COMPUTER AIDED DESIGN of the HARRISON SINGLE PIVOT GRASSHOPPER ESCAPEMENT GEOMETRY

Computer Aided Design of the Harrison Single Pivot Grasshopper Escapement Geometry

This publication replaces 'Perfecting the Harrison Single Pivot Grasshopper Escapement', published in 2009 by the same author. As a consequence of recent discoveries within Harrison's illustrated work, 'mean end/start ratio' replaces mean torque arm ratio and corrections for varying forces are no longer applied. Mathematical modelling is displaced by a more versatile and capable graphical construction, most accurately (although not essentially) created using widely available Computer Aided Design (CAD) software.

1st Edition, 21st September 2011

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PREFACE

John 'Longitude' Harrison (1693-1776) created his first grasshopper escapement with the simple objectives of eliminating sliding friction, the detrimental effects of wear and the unpredictable inconsistencies of lubrication. He later discovered that versatile impulse characteristics and a capacity for high escaping arcs could also be manipulated to his advantage, as he strove to extract unprecedented performance from entirely mechanical timekeepers exposed to the Earth's atmosphere. Perhaps unintentionally, Harrison also created one of the most beautiful and mesmerising mechanical escapements ever devised.

After almost two and a half centuries of neglect, the single and twin pivot grasshopper escapements have recently enjoyed something of a revival, the motive in almost all cases being an understandable desire to reproduce the fascinating 'kicking grasshopper' motions, from which the escapement derives its popular name. Sadly, the remarkable performance advantages and the means by which they should be achieved have been almost universally overlooked, ignored or misunderstood. As a consequence, recent history is littered with hideous grasshopper mutations, more likely to turn Harrison in his grave than permit the escape wheel to rotate as he intended. There can, however, be little argument that Harrison's own failure to record a clear explanation of his invention has been responsible for a great deal of the neglect and ignorance.

In an attempt to resolve this sorry and unnecessary situation, 'Computer Aided Design of the Harrison Single Pivot Grasshopper Escapement Geometry' will gather, analyse, interpret and explain every one of Harrison's recorded intentions for the performance of his astonishing invention and offer a straightforward technique for the simultaneous incorporation of every one of them within escapement geometries of the single pivot configuration.

David Heskin, Lancashire, England, September 2011

INTRODUCING THE SINGLE PIVOT GRASSHOPPER ESCAPEMENT

MECHANICAL ARRANGEMENT

Figure 1 illustrates the assembled components of a Harrison 'single pivot' grasshopper escapement, drawn to no particular scale, rigidly connected to a greatly shortened, symbolic representation of a suspended simple pendulum. In comparison with common escapements, such as the anchor and dead beat, the mechanical arrangement is slightly more involved, rather unusual and far more interesting. Most striking are two wooden arms, pivoting about a single pin and an exceptionally large escape wheel, with many teeth. Alternative configurations, escape wheel sizes and tooth counts are possible, although Harrison apparently identified the illustrated escapement as one of two ideal solutions for land-based precision regulators incorporating seconds-beating pendulums.

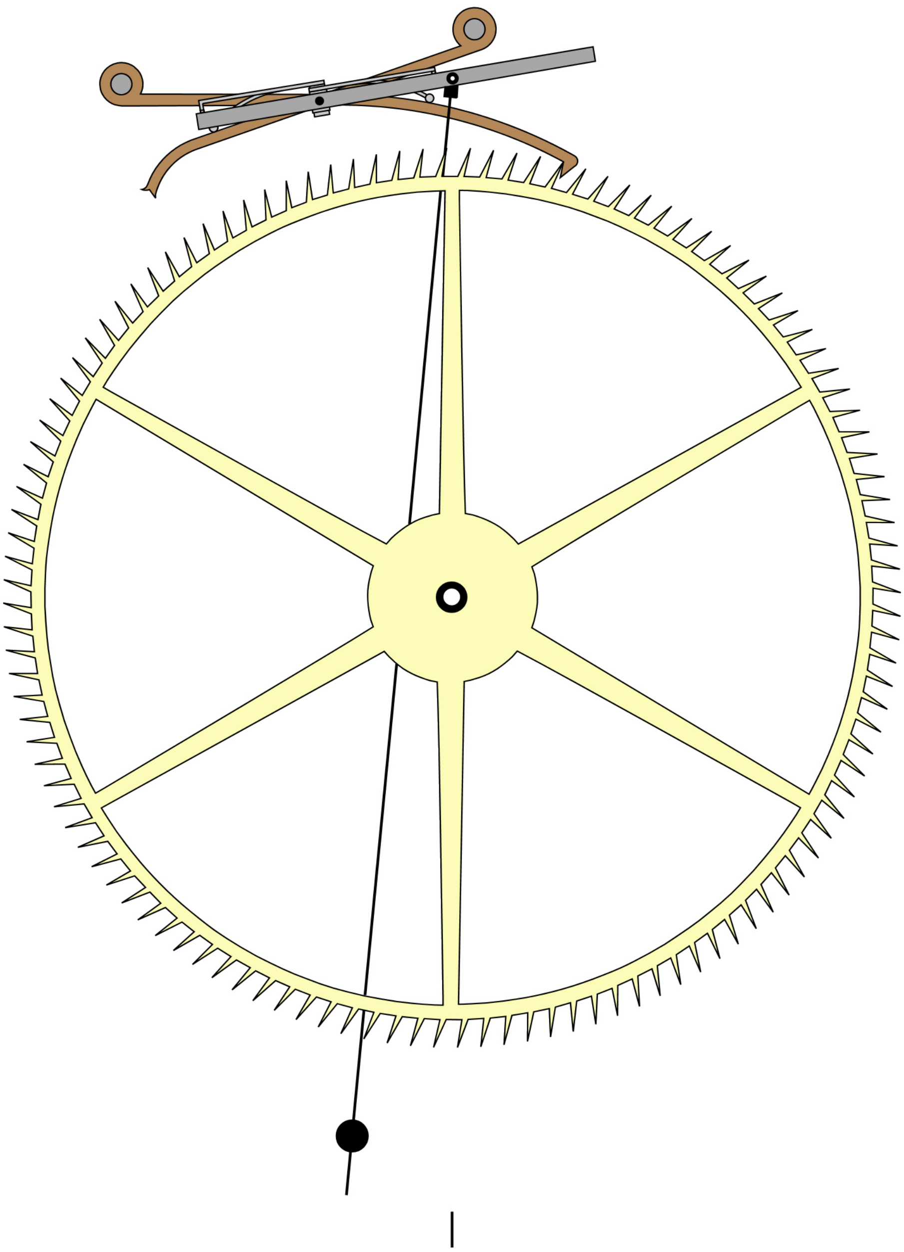


Figure 1 - Harrison single pivot grasshopper escapement

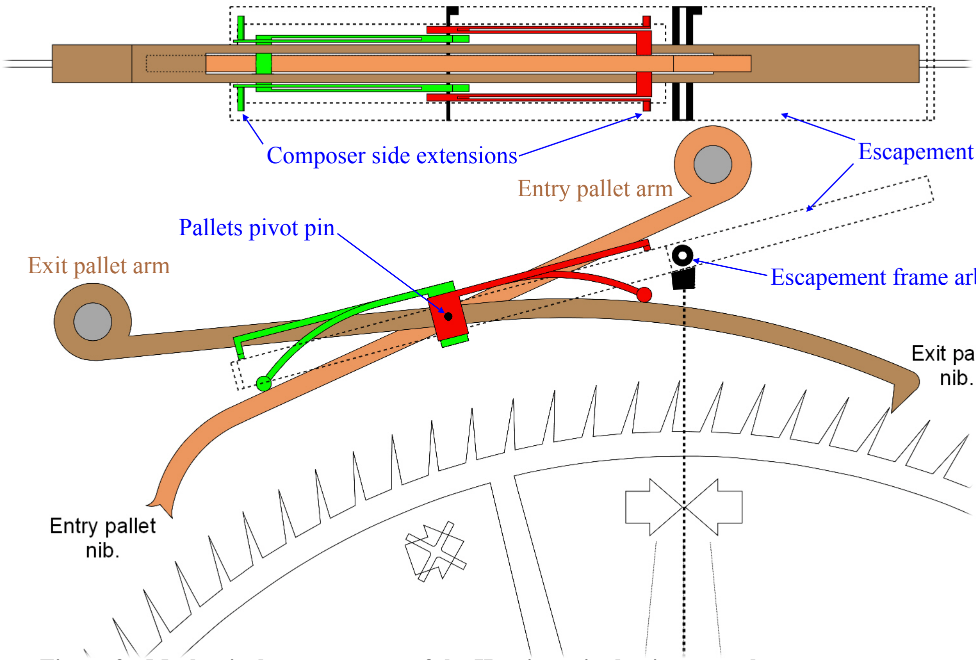


Figure 2 - Mechanical arrangement of the Harrison single pivot grasshopper escapement

Figure 2 illustrates the escapement of Figure 1 in more detail, viewed from above (upper illustration) and from the assumed front (lower illustration).

In the interests of simplicity, the pendulum is symbolically represented as a broken black line, in this case vertical, rigidly attached to the **'escapement frame'**. The escapement frame, typically of metal, such as brass, is illustrated in Figure 2 as transparent, with a broken outline, whilst **Figure 3** shows it to a reduced scale as a solid white object. The circular, saw-toothed, brass **'escape wheel'**, which in all illustrations rotates clockwise about its axis during normal operation, is only partially represented in Figure 2 and is illustrated as transparent.

Pallet **'nibs'** are formed at the ends of two hardwood **'pallet arms',** shown in Figure 2 in contrasting brown colours, for ease of explanation. The **'entry pallet nib'** is that end of the lighter brown **'entry pallet arm'** met by the tip of an escape wheel tooth as the tooth rotates clockwise towards the escapement assembly. The **'exit pallet nib'** is that end of the darker brown **'exit pallet arm'** left behind by an escape wheel tooth tip as the tooth rotates clockwise away from the escapement assembly. Figures 2 and 3 show how the entry pallet arm pierces the exit pallet arm, there being sufficient clearance to avoid friction between the two. Each pallet arm is mounted freely and completely independently upon a shared, single **'pallets pivot pin'**, which is affixed rigidly (i.e. with no freedom to rotate or slide) to the escapement frame. The origin of the name **'single pivot grasshopper escapement'** should now be clear. The pallet arms are weighted at their **'tail'** ends, usually with metal inserts (shown as solid grey circles), such that both arms are tail-heavy, for reasons to be explained shortly

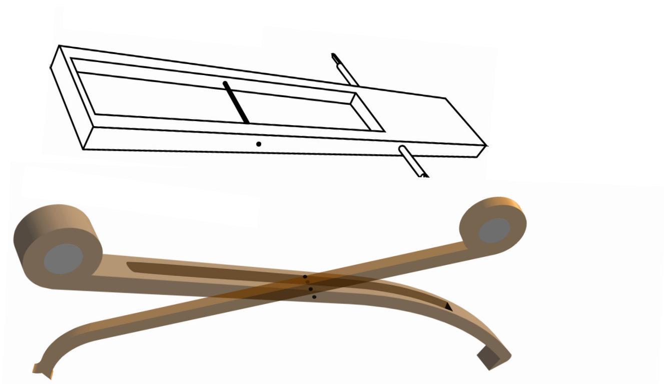


Figure 3 - Partially exploded view of escapement frame, single pallets pivot pin, escapement frame arbor and tail-heavy pallet arms. Composers, pendulum, crutch and escape wheel not included.

Also mounted freely and completely independently upon the single pallets pivot pin are two components commonly referred to as **'composers'**. In Figure 2, the **'entry composer'** is shown as green and the **'exit composer'** is shown as red. Both composer are very obviously nose-heavy, the **'nose'** being the free end, furthest from the single pivot pin. Each composer nose incorporates two tiny **'side extensions'**, also called **'stops'**. When the composer stops contact the escapement frame (as shown in Figure 2), the composer noses can move no further downwards. The composers are then said to be **'resting'** upon the escapement frame. Detailed explanations of the behaviour of the composers and their interactions with the escapement frame and pallet arms will follow shortly.

Labelled in Figure 2 and clearly illustrated in **Figure 3**, the escapement frame is rigidly attached to what is commonly referred to as the 'pallet arbor', but will more appropriately be called the **'escapement frame arbor'** for the grasshopper escapement. A conventional crutch (not illustrated, in the interests of simplicity) would normally be rigidly affixed to the escapement frame arbor, transmitting torque (turning effort) from the escapement to the independently suspended pendulum. The escapement frame arbor may, therefore, sometimes be referred to as the **'crutch arbor'**. Each end of the steel escapement frame arbor incorporates an almost frictionless **'knife edge pivot'**, consisting of a small, sharp, hardened V-section rocking within a sharp, polished, axial V-shaped groove in the upper face of glass plate support (the latter not illustrated).

COMPONENT BEHAVIOUR

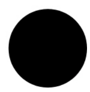
This section will begin with simple descriptions of the behaviour of individual components and will end with detailed explanations of their various interactions.

KEY TO ILLUSTRATIONS

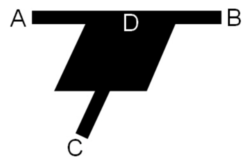
Unless stated otherwise, all views are from the assumed front of an imaginary timekeeper movement. From that viewpoint, all escape wheels are assumed to normally rotate clockwise.

'Anchored pivot' (greatly enlarged).
A pivot attached to the Earth.
Movement of the pivot is not possible.
Free rotation about the pivot is possible.

'Travelling pivot' (greatly enlarged).
Rigidly incorporated within a moveable object.
Can only move in perfect unison with the object.
Free rotation about the pivot is possible.



'Rigid attachment' (greatly enlarged).
Joins objects rigidly together.
In this example, objects A, B and C are rigidly joined at D.



Symbolic pushing hand (greatly enlarged).
Indicates that a component is being pushed (with undefined force, unless stated) in the indicated direction by an imaginary, extremely small operator.
Considerably smaller than the average human hand.

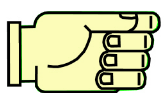


Figure 4 - This is a view from the assumed front of the single pivot grasshopper escapement, with almost all of the components removed. The escapement frame (now shown as a grey rectangle) is illustrated edge-on, pivoting freely, along with the rigidly attached escapement frame arbor, about the fixed escapement frame arbor axis. The pendulum rod is symbolically represented as a bold, broken black line, vertical in this instance, rigidly connected to the escapement frame. In reality (although not illustrated, in the interests of simplicity), the pendulum would be independently suspended and rigidly linked to the escapement frame via a crutch. The symbolic pushing hands to either side of the pendulum indicate that the pendulum and rigidly attached escapement frame are being held stationary in the illustrated position by a very small assistant.

For future reference, the minimum *theoretical* pendulum angular displacement for correct escapement operation is indicated by two short, thin, broken lines, symmetrically disposed to the left and right of the vertical position. Both lines radiate from the escapement frame arbor axis, about which the pendulum and rigidly attached escapement frame pivot in unison. In practice, the driving weight (or, alternatively, the coiled spiral spring) of any timekeeper must be adjusted to supply a small excess of energy to the escapement, to guarantee reliable, continuous operation, despite unavoidable and unpredictable influences and disturbances. The excess energy generates pendulum displacements slightly beyond the two illustrated lines. A third thin, broken black line, almost entirely obscured by the pendulum rod, but clearly visible in the next illustration, coincides with the vertical position of the pendulum.

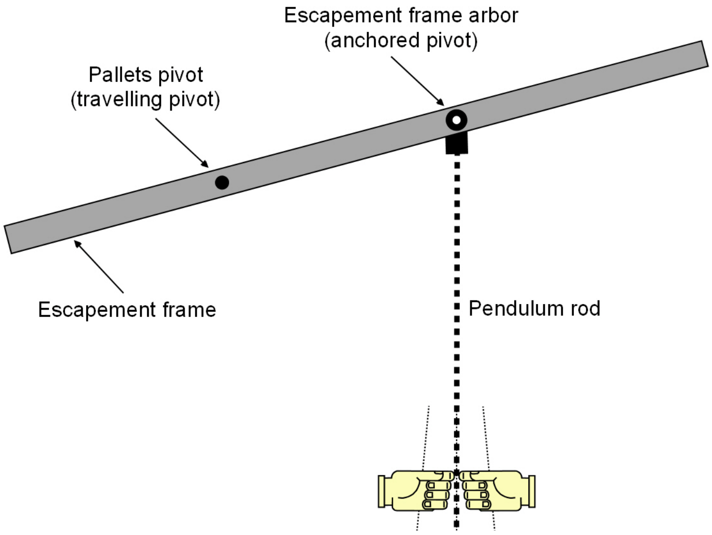


Figure 4 - Escapement frame and pendulum in isolation. Pendulum held in the vertical position.

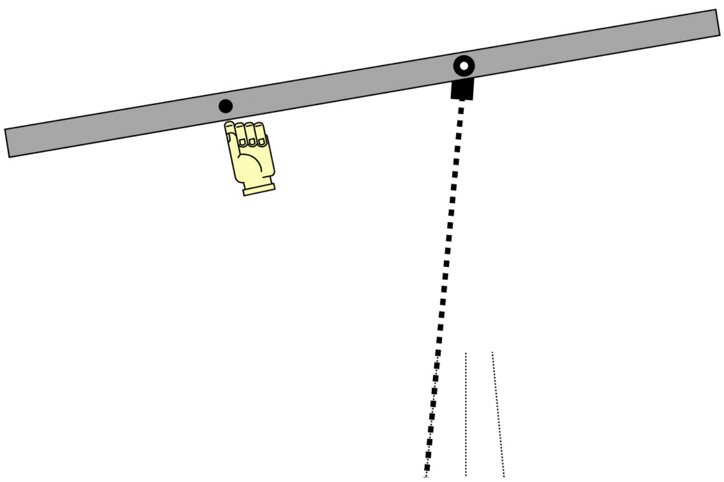


Figure 5 - Clockwise torque applied to the escapement frame arbor.

Figure 5 - Demonstrates the effect of releasing the pendulum rod and applying and maintaining clockwise torque (turning effort) about the escapement frame arbor axis, generated by the symbolic upwards pushing hand positioned (merely for convenience of illustration and explanation) near the single pallets pivot pin.

The rigidly connected pendulum, escapement frame, escapement frame arbor and single pallets pivot pin have all rotated clockwise, in unison, about the escapement frame arbor axis. The movement is resisted by the Earth's gravity, acting vertically downwards through the centre of gravity of the assembly. When the applied torque and the resistance due to gravity are in balance, the pendulum will maintain a stationary position to the left of vertical, as illustrated. In this example, the pendulum is shown aligned with its left-hand displacement marker for convenience of illustration and for no other reason.

In normal escapement operation, the torque required to maintain continuous motion of the pendulum need only be sufficient in magnitude to make up for small energy losses to air resistance, pendulum suspension etc. and the unavoidable and unpredictable influences and disturbances mentioned earlier. It should therefore be bourne in mind that in Figure 5 (and up to and including Figure 20), normal operating torques and forces will often be appreciably lower than the illustration might suggest.

Figure 6 - Demonstrates the effect of releasing the pendulum rod and applying and maintaining anticlockwise torque about the escapement frame arbor axis, generated by the symbolic downwards pushing hand near the single pallets pivot. The pendulum, escapement frame, escapement frame arbor and single pallets pivot pin have all rotated anticlockwise, in unison, about the escapement frame arbor axis and the pendulum has swung to the right of vertical until a balance of applied torque and resistance due to gravity has been established, when all motion ceases.

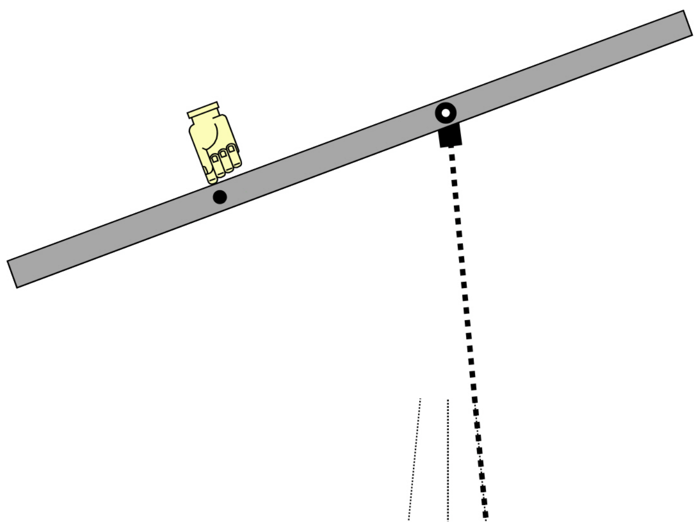


Figure 6 - Anticlockwise torque applied to the escapement frame arbor.

Figure 7 - Both composers have been added to the previous configuration. The entry composer is shown in green, the exit composer is shown in red and the escapement frame is now shown as a transparent rectangle with a broken outline. The pendulum is being held in the vertical position, in the interests of straightforward explanation.

Recall that the composers are nose-heavy. Thus, the entry composer (green) is at all times inclined to rotate anticlockwise about the single pallets pivot pin, whilst the exit composer (red) is at all times inclined to rotate independently clockwise about the single pallets pivot pin. The composer stops (side extensions) are illustrated as tiny square elements at the ends of the upper, straight composer limbs. Two broken circles highlight the points of contact between the composer stops and the escapement frame. Further anticlockwise (entry composer) and clockwise (exit composer) rotations of the nose-heavy composers about the single pallets pivot pin are prevented by contact between the stops and the escapement frame. In the absence of any forces other than gravity, both composers are obliged to 'rest' in the illustrated positions.

Note that, although the composers share the single pallets pivot pin, they each pivot entirely independently about it. At no point in the entire escapement cycle will there be any direct interaction between the two composers.

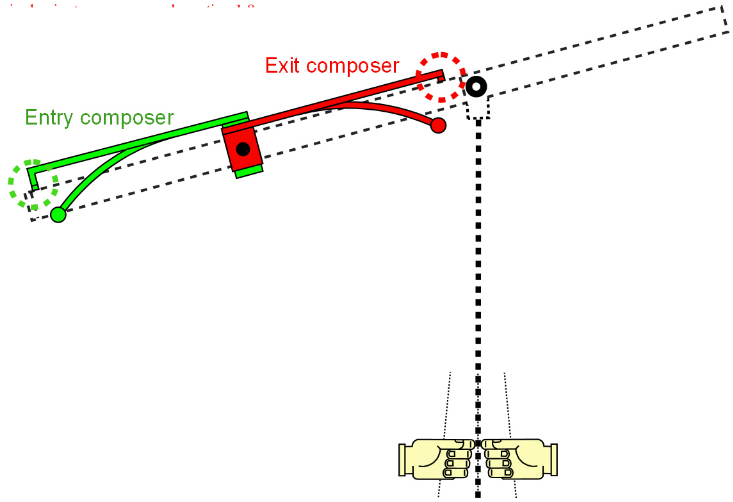


Figure 7 - Both nose-heavy composers <u>independently</u> 'resting' upon the escapement frame.

Figure 8 - Up to this point, the composers have been shown as green (entry) and red (exit) to emphasise their independent forms and positions. In Figure 8 and all subsequent illustrations, red or green will only be adopted when the applicable composer is away from its 'resting' position. **Any 'resting' composer will henceforth be grey in colour.**

In Figure 8, the pendulum and rigidly attached escapement frame are being held stationary, with the pendulum vertical. The noses of both composers have been raised by entirely separate external forces, generated by the pushing hands of a second assistant. For clarity, the illustrated displacement of both composers is considerably greater than should ever be the case during normal escapement operation. The entry composer stops have broken contact with the escapement frame and the entry composer is now generating an independent, continuous, anticlockwise resistance to the applied force. The exit composer stops have likewise broken contact with the escapement frame and the exit composer is now generating an independent, continuous, clockwise resistance to the separately applied force.

The rotation of the entry composer about the single pallets pivot pin is entirely independent of the rotation of the exit composer about the single pallets pivot pin and vice versa. Thus, for example, if the hand input to the entry composer was to be removed (not illustrated), the entry composer would return to its resting position and its colour would change from green to grey. The exit composer, still subjected to an upward force, would independently remain in the raised position and still be coloured red.

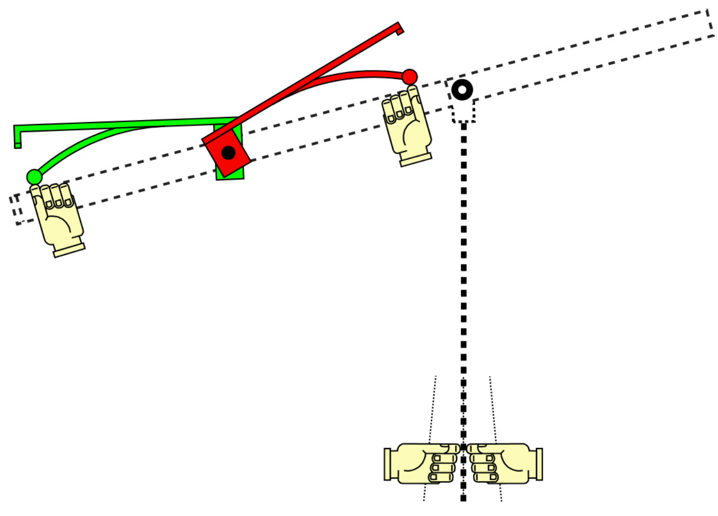


Figure 8 - Composers <u>independently</u> raised (with exaggerated displacements).

Figure 9 - Both (brown) hardwood pallet arms have been added to the previous configuration. The pallet arm 'tails', are weighted with (grey, circular section) metal inserts such that both arms are tail-heavy. The entry pallet arm is, therefore, continuously inclined to rotate clockwise about the single pallets pivot pin, which is currently holding it in continuous contact with the tiny cylindrical (circular cross-section) feature at the nose end of the lower, curved limb of the entry composer. Most importantly, the clockwise entry pallet arm torque about the single pallets pivot pin is arranged to be less than the independent anticlockwise torque generated by the entry composer about the single pallets pivot pin. Therefore, the entry pallet arm and entry composer *combination* generates an overall excess of anticlockwise torque, which holds the entry composer stops in contact with the escapement frame. The exit pallet arm and exit composer are likewise weighted, such that the tail-heavy exit pallet arm is held against the more nose-heavy exit composer, whilst the exit pallet arm and exit composer *combination* generates an overall excess of clockwise torque, holding the exit composer stops in contact with the escapement frame. Note carefully that despite the sharing of the single pallets pivot pin, the behaviour of any entry component is at all times entirely independent of the behaviour of any exit component and vice versa.

As a useful aside, the illustrated mass of the escapement frame to the right of the escapement frame arbor will obviously fail to balance the greater combined mass of the pallet arms, composers, pivot pin and escapement frame to the left of the arbor. Balance could be achieved by adding mass to the right-hand side of the escapement frame, although perfect balance will henceforth be assumed, but not illustrated.

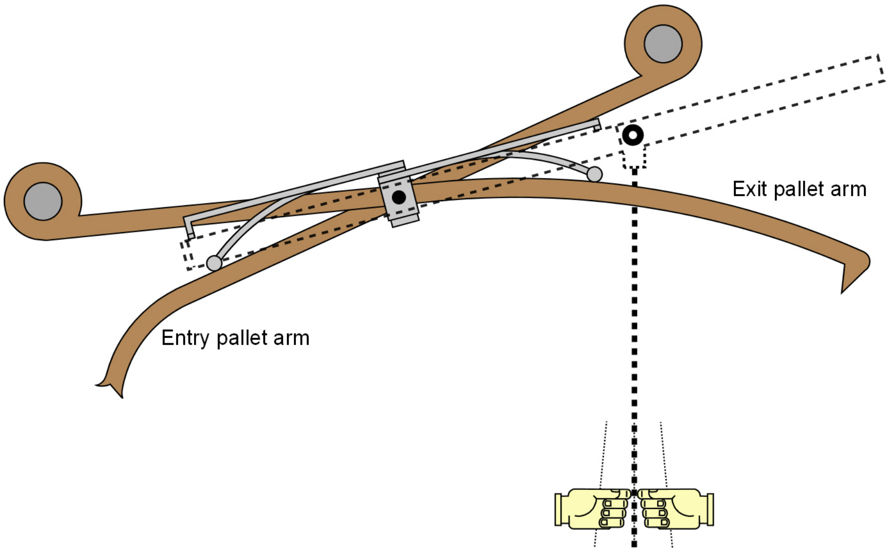


Figure 9 - Tail-heavy pallet arms holding themselves against their paired composer.
Composer stops held against escapement frame by more nose-heavy composers.

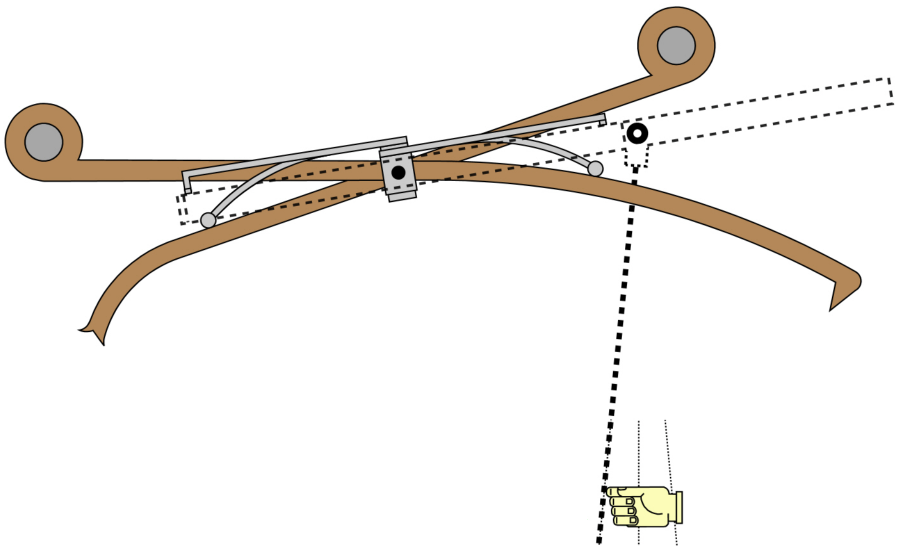


Figure 10 - Pendulum rod pushed to the left.

Figure 10 - The pendulum rod has been pushed and held to the left of vertical, obliging the rigidly attached escapement frame, escapement frame arbor and single pallets pivot pin, together with the pivoted pallet arms and composers, to rotate *en masse* clockwise through the same angle about the escapement frame arbor. No component has moved relative to any other component. Thus, each tail-heavy pallet arm will remain in contact with its paired composer and each nose-heavy composer/arm pairing will continue to rest in the illustrated positions, with the composer stops in contact with the escapement frame.

Figure 11 - The pendulum rod has been pushed and held to the right of vertical, obliging the rigidly attached escapement frame, escapement frame arbor and single pallets pivot pin, together with the pivoted pallet arms and composers, to rotate *en masse* anticlockwise through the same angle about the escapement frame arbor. As in the previous situation, no component has moved relative to any other component.

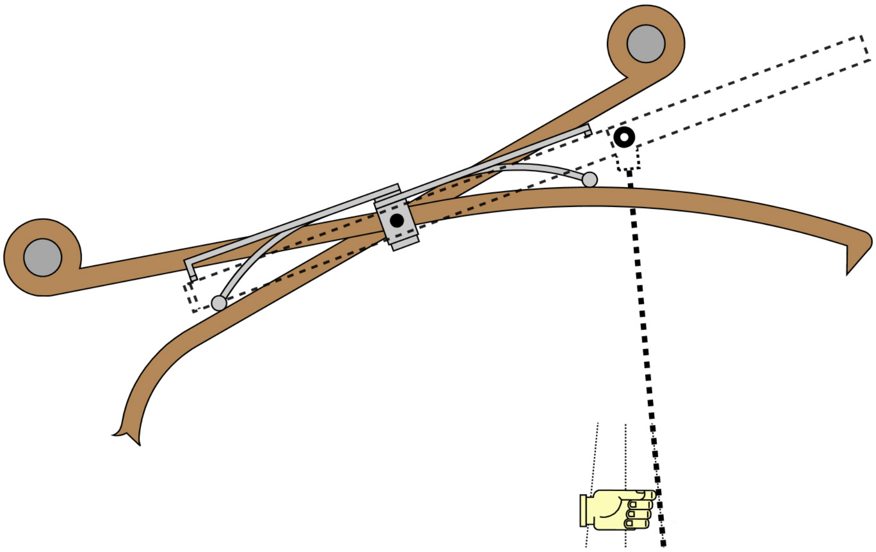


Figure 11 - Pendulum rod pushed to the right.

Figure 12 - Sufficient, separate, upwards forces have been applied to the nib ends of each pallet arm. The entry pallet arm has rotated clockwise about the single pallets pivot pin and has lifted the entry composer, which changed from grey to green as soon as its stops departed the resting position. Completely independent of entry behaviour, the exit pallet arm has rotated anticlockwise about the single pallets pivot pin and has lifted the exit composer, which changed from grey to red as soon as its stops departed the resting position

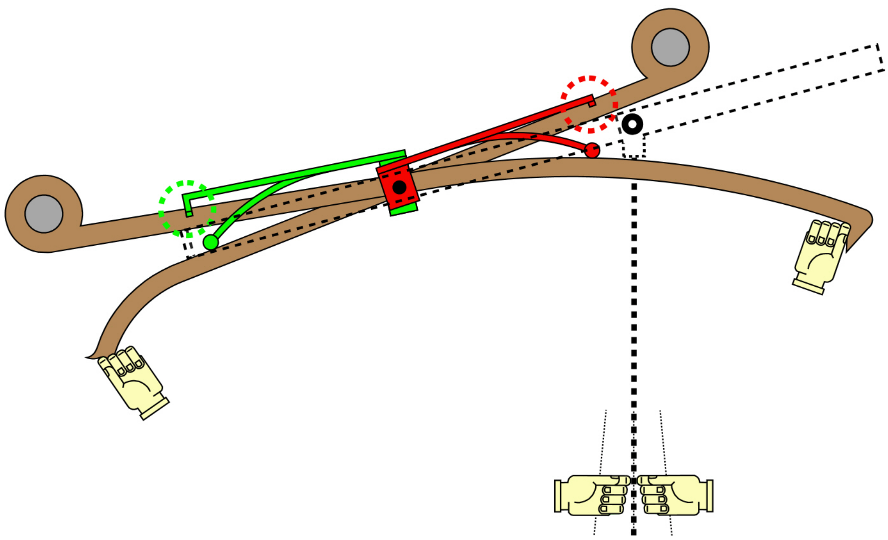


Figure 12 - Completely independent consequences of separately raising the nib end of each pallet arm.

Figure 13 - Separate downward forces have been applied to the nib ends of each pallet arm. Sufficient force has overcome the clockwise torque generated by the tail-heavy entry pallet arm, which has therefore rotated anticlockwise about the single pallets pivot pin. Since the entry composer can rotate no further anticlockwise than its resting position, it has been left behind, with its stops resting upon the escapement frame. Separation of the entry pallet arm from the entry composer is emphasised by the broken green circle. Entirely independent of entry component behaviour, the exit pallet arm has rotated clockwise about the single pallets pivot pin and the exit composer has been left behind, in its resting position. Exit pallet arm and composer separation is emphasised by the broken red circle.

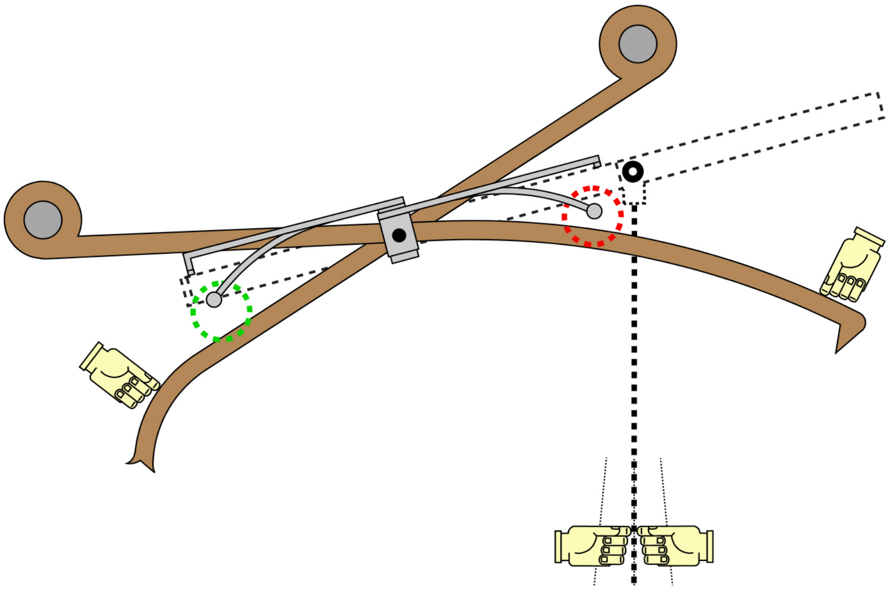


Figure 13 - Completely independent consequences of separately depressing the nib end of each pallet arm.

STARTING THE GRASSHOPPER

This section will describe the sequence of operations required to ***safely*** set the single pivot grasshopper escapement in motion, beginning with an unwound movement and a vertical pendulum, all completely at rest. The grasshopper escapement demands particular care during the starting procedure, in an effort to avoid complete, simultaneous detachment of both pallet nibs from all of the escape wheel teeth tips (referred to herein as escapement '**trip**'), permitting free-running of a driven escape wheel (referred to herein as escape wheel **'runaway'**). Because one of the purposes of any movement train is to increase the rate of rotation of the escape wheel relative to the weight barrel, the teeth of a free and driven escape wheel could accelerate rapidly to a high terminal speed. Any subsequent contact between speeding escape wheel teeth and either escapement pallet nib could all too easily result in damage, potentially considerable. Further damage and/or injury might also arise from the continuous fall of the driving weight, possibly culminating in severe impact with the bottom of the clock case and/or an unwary operator. Despite those warnings, care must be taken to avoid unfair conclusions. Conditional upon sound design, construction, setup and operation, Harrison's grasshopper escapement is one of the most dependable, consistent, versatile, maintenance-free and astonishingly durable mechanical escapements ever devised. *It does not, however, suffer fools at all gladly.*

Figure 14 - Represents the complete escapement, with the escapement frame hereafter illustrated as a grey rectangle, rigidly connected to the pendulum. Below the escapement frame assembly lies the escape wheel, which is normally driven clockwise about the escape wheel arbor axis, out of illustrated view at the centre of the wheel. The crossed arrow within the rim of the escape wheel indicates that it is currently stationary and a note confirms that no torque is being applied to the escape wheel arbor. Since, as stated earlier, all illustrations assume perfect balance of the entire assembly, the pendulum would adopt a vertical position when free, although the pendulum is illustrated with restraining hands, to avoid any doubt that it is stationary and vertical.

Of considerable significance are the length and position of the exit pallet nib. The broken red circle highlights the exit nib and the closest escape wheel tooth anticlockwise removed from it. If a clockwise torque was to be applied to the escape wheel arbor, the escape wheel would rotate freely clockwise until the highlighted escape wheel tooth encountered the exit nib, at which point escape wheel runaway would be prevented, albeit somewhat precariously. Contact obviously relies upon adequate exit nib length, as will be explained in more detail in due course.

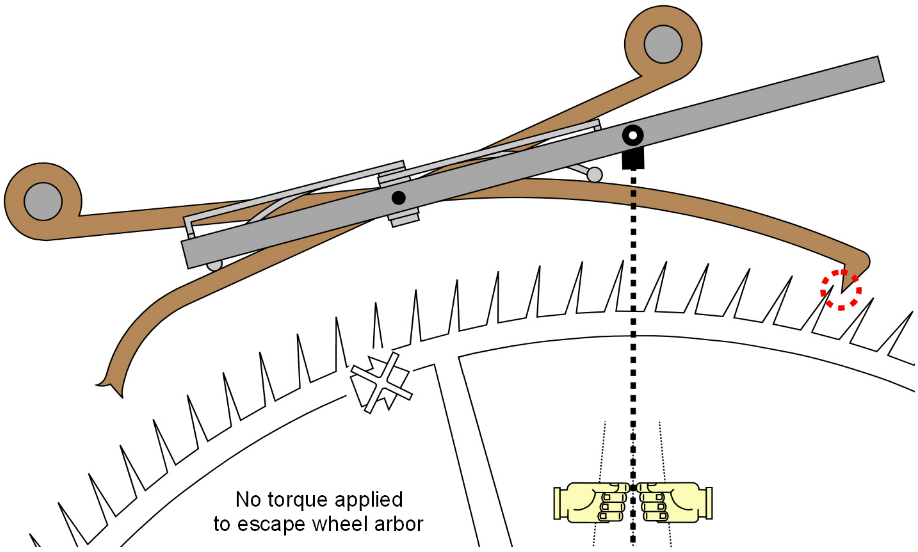


Figure 14 - Escapement, escape wheel and pendulum at rest. No escape wheel torque. Pendulum vertical.

Another potential cause of escapement trip must also be avoided. Beginning with the situation shown in Figure 14, should the pendulum be displaced to the right of vertical (not illustrated), the entire escapement frame assembly (the frame, arbor, single pivot, both pallet arms and both composers) would rotate, en masse, anticlockwise about the escapement frame arbor axis, which would lift the exit pallet nib away from the escape wheel. Given sufficient pendulum displacement, the exit nib would eventually be lifted completely clear of the circular sweep of the escape wheel teeth tips and the escape wheel would thereafter be free to runaway if torque was to be applied. Subsequent pendulum return to the left (including the complete release of the pendulum) would permit potentially damaging contact between the runaway escape wheel teeth and the exit nib. Assuming adequate exit pallet nib length in the first place, it is therefore essential that the pendulum is not displaced towards the exit side of the escapement (i.e. to the right in this case).

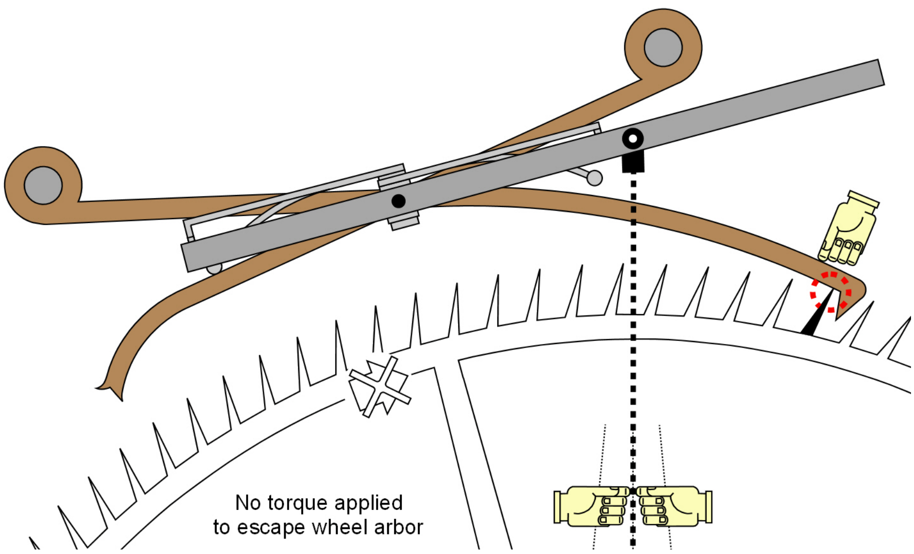


Figure 15 - Underside of exit pallet arm gently depressed onto an escape wheel tooth tip.

Figure 15 - The 'at rest' scenario of Figure 14 is duplicated, with the exception that the exit pallet arm has been *gently* depressed near the nib end, until the underside of the arm *very lightly* contacts an escape wheel tooth tip, as highlighted by the broken red circle. As the exit pallet arm is lowered, it will separate from the exit composer, which must remain in its resting position. A black infill highlights the escape wheel tooth in contact with the exit pallet arm. The *gentle* manual input must be maintained until release is instructed.

An alternative, possibly more convenient, but arguably less secure approach to depressing the exit nib directly, would be to displace the pendulum towards the entry side (i.e. to the left in this case) until the underside of the resting exit pallet arm contacted the escape wheel. The pendulum must be held in that position, although, subject to weightings, anywhere slightly further to the left would be easier and no less acceptable, since the exit composer would merely be lifted from its resting position, thereby accommodating any excess exit pallet arm movement. This option will be explained no further, although Figures 16 to 20 inclusive illustrate valid continuations.

Figure 16 - The note within the illustration confirms that torque has been applied to the escape wheel, by winding the timepiece. Provided that the manual depression of the exit nib (as per Figure 15) is not excessive, the tip of the black escape wheel tooth will slide along the underside of the exit pallet arm until arrested at the corner of the exit nib, as highlighted by the broken red circle. The highlighted corner will henceforth be referred to as the exit nib '**locking corner**'. The gentle manual force depressing the exit pallet nib may now be removed, as illustrated, subject to suitable choices of escape wheel and pallet nib materials and sufficient escape wheel torque. Sufficient static friction between the driven, black escape wheel tooth tip and the exit pallet nib locking corner will thereafter hold the exit nib in the illustrated position, overcoming the tail-heaviness of the exit pallet arm. This situation will henceforth be referred to as nib locking corner **'capture'**. If adequate escape wheel torque is maintained, the situation of Figure 16 will arguably be more secure than that of Figure 14, if only by virtue of deeper exit nib engagement. Nevertheless, in view of the potentially disastrous consequences of escapement trip and escape wheel runaway, a cautious operator might wish to maintain gentle depression of the exit pallet nib, thereby guaranteeing secure escape wheel restraint. The precaution will not be illustrated, although there will be a reminder when all manual inputs must be removed.

As an aside, pallet arms of a suitable hardwood are well suited to this purpose, whereas lower friction materials, such as metal, some plastics or 'greasy' woods such as lignum vitae would generate lower static friction, resulting in less secure capture, or a failure to capture. Although increased escape wheel torque might eventually compensate, it might also transmit an undesirable excess of energy to the pendulum. Also of passing interest, if the vertical restraint of the pendulum was to be released, it would merely adopt a stationary position very slightly anticlockwise from vertical, as clockwise escape wheel torque feeds through the exit pallet arm, single pivot pin, escapement frame, escapement frame arbor and crutch, becoming anticlockwise in the process.

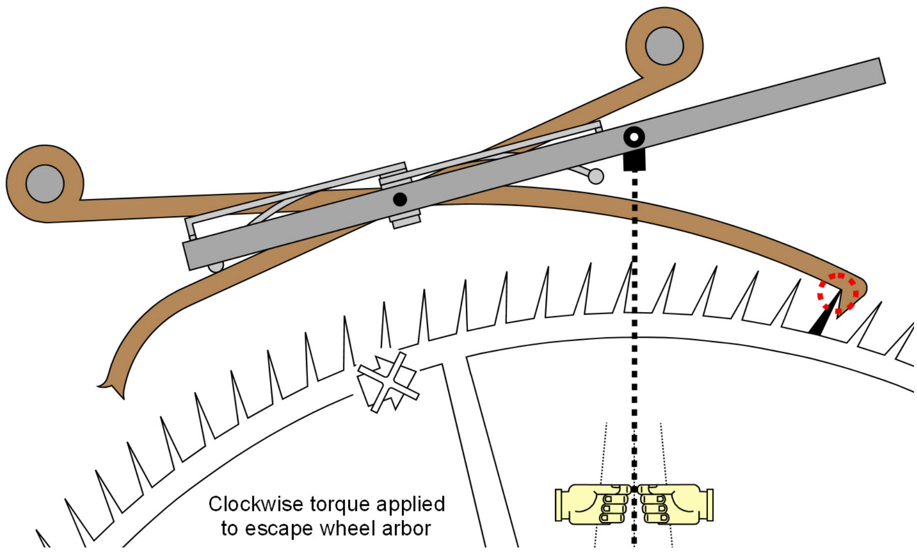


Figure 16 - Torque applied to the escape wheel. Exit pallet nib locking corner captured by static friction. Pendulum held stationary in the vertical position (although no longer essential).

Figure 17 - This illustration represents a situation in which several motions are imperceptibly *beginning* to occur. Our assistant begins to push the pendulum to the right and the rigidly attached escapement frame and single pivot pin begin to rotate anticlockwise about the escapement frame arbor, as indicated by the grey arrow. Those motions induce clockwise rotation of the exit pallet arm about the single pallets pivot pin, as the captured exit pallet nib locking corner pivots about the black escape wheel tooth tip (emphasised by the small red circle). The escape wheel, to which torque is still being applied, begins to rotate clockwise, as it transmits its energy, in sequence, to the exit pallet nib locking corner, exit pallet arm, single pallets pivot pin, escapement frame, escapement frame arbor, crutch (not illustrated) and pendulum. The resting entry pallet arm begins to rotate anticlockwise about the escapement frame arbor (<u>not</u> about the single pallets pivot pin), which begins to swing the entry nib closer to the escape wheel.

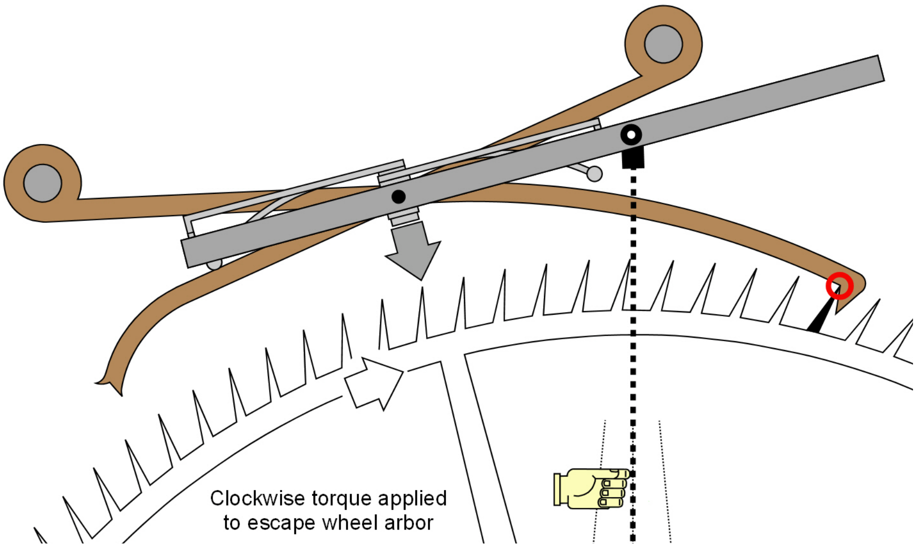


Figure 17 - Imperceptible commencement of manual pendulum displacement to the right.

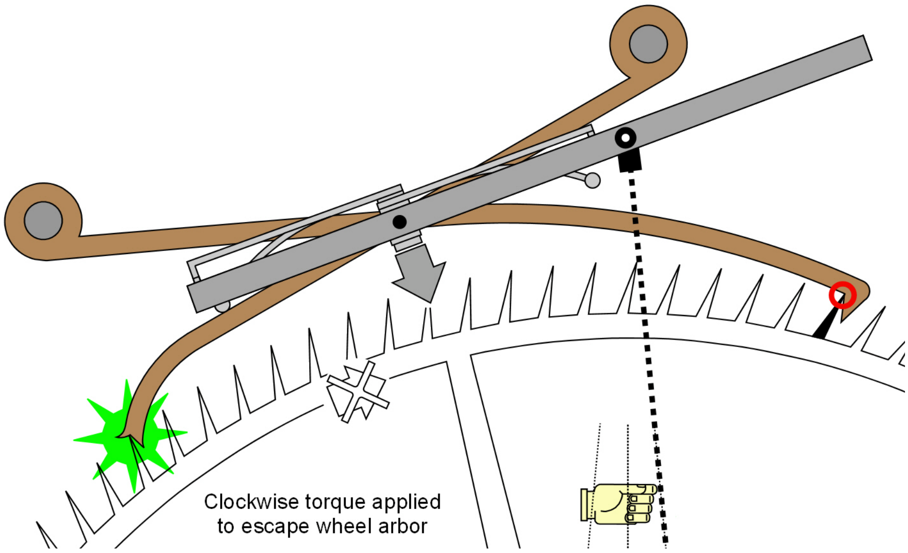


Figure 18 - Pendulum coincides with right hand displacement marker.
Entry pallet nib locking corner contacts an escape wheel tooth tip. Escape wheel halted.

Figure 18 and Figure 19 - These separate figures represent several events occurring virtually simultaneously, but most clearly explained in two separate stages. Any precautionary manual input to the exit pallet nib, mentioned at the end of the Figure 16 explanation, must be removed just before these events.

Figure 18 - Our assistant has continued to push the pendulum to the right, rotating the escapement frame and single pallets pivot pin further anticlockwise about the escapement frame arbor. The exit pallet arm, still captured at the exit nib locking corner by the black escape wheel tooth tip, rotates further clockwise about the single pallets pivot pin. The entry pallet nib, which has been swinging anticlockwise about the escapement frame arbor, has just contacted the escape wheel. The geometry must be designed such that contact between the entry pallet nib and the appropriate escape wheel tooth tip occurs precisely at the nib locking corner at the instant the pendulum coincides with the right-hand displacement marker (less precise contact could result in forbidden sliding friction and wear between the tooth tip and the entry nib and could even lead to potentially damaging escapement trip and escape wheel runaway). Precise contact between the appropriate tooth tip and the entry nib locking corner immediately prevents further clockwise rotation of the escape wheel. Nib and escape wheel materials aside, there may or may not be an audible 'click' on contact, subject to the energy imparted to the pendulum.

Figure 19 - The pendulum has been pushed further to the right by an imperceptibly small amount. The single pallets pivot pin continues to rotate anticlockwise about the escapement frame arbor, obliging the captured entry pallet arm to rotate clockwise about the single pallets pivot pin. Such motions can only be accommodated by forcing the escape wheel into recoil. As another consequence of entry pallet arm rotation, the entry composer stops are imperceptibly lifted away from the escapement frame (entry composer colour changes from grey to green).

At the instant escape wheel recoil begins, escape wheel impulse to the exit pallet nib locking corner is removed and static friction between it and the black escape wheel tooth tip is lost. This event represents the second of three phases through which each side of the grasshopper escapement cycles, referred to herein as **'release'**, whereby a previously captured pallet nib locking corner is released from captivity by a recoiling escape wheel tooth tip. The released, tail-heavy exit pallet arm rotates anticlockwise about the single pallets pivot pin and, after a brief period of free motion, encounters the resting (grey) exit composer. Subject to the degree of tail weighting and inertia of the exit pallet arm, the motion and encounter could be anything from slow, gentle and virtually silent, to rapid and hard, generating an audible 'click'. There may even be bouncing of the pallet arm upon contact, generating one or more 'clicks' of diminishing volume. In extreme cases and/or with a weak composer, contact could induce bounce of the composer on the escapement frame, perhaps creating sounds. Note carefully that, unlike common escapements such as the anchor and dead beat, the described sounds are not immediately created by contact between an escape wheel tooth tip and a pallet nib locking corner and would, therefore, be of limited value for setting the timekeeper in beat.

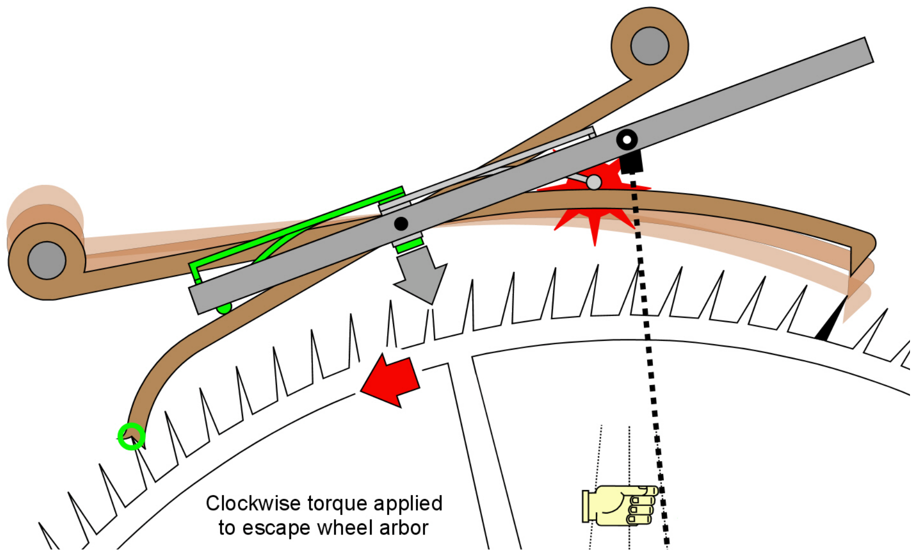


Figure 19 - Virtually coincident with Figure 18.
Entry pallet nib locking corner captured. Entry composer raised. Escape wheel recoil begins.
Exit pallet arm released and arrested by exit composer.

Figure 20 - Pendulum displacement to the right has continued slightly beyond the right-hand marker. In normal operation, with all operator inputs removed, the extent of the additional displacement is adjusted by altering the driving weight, the objective being to ensure reliable escapement operation, despite unavoidable and unpredictable variations in external and other influences. Although escapement **'supplementary arc'** is the correct term, it will be more descriptive to refer to the additional motion as pendulum **'overswing'** (Harrison referred to it as 'overplus'). Overswing is the third phase of operation of the grasshopper escapement. During the illustrated pendulum overswing to the right, the entry pallet arm has been forced to rotate further clockwise about the single pallets pivot pin, which has lifted the entry composer stops further away from the escapement frame (more clearly shown in the magnified inset). As a consequence of those motions, the escape wheel has been obliged to recoil further anticlockwise (indicated by the red arrow).

From the limit of overswing to the right, the withdrawal of all manual inputs would mark the completion of the escapement start-up procedure and the simultaneous commencement of continuous escapement cycling, sustained entirely by the raised driving weight (or coiled spiral spring) of the timekeeper. Figures 21 to 32 inclusive will explain that escapement cycle in detail.

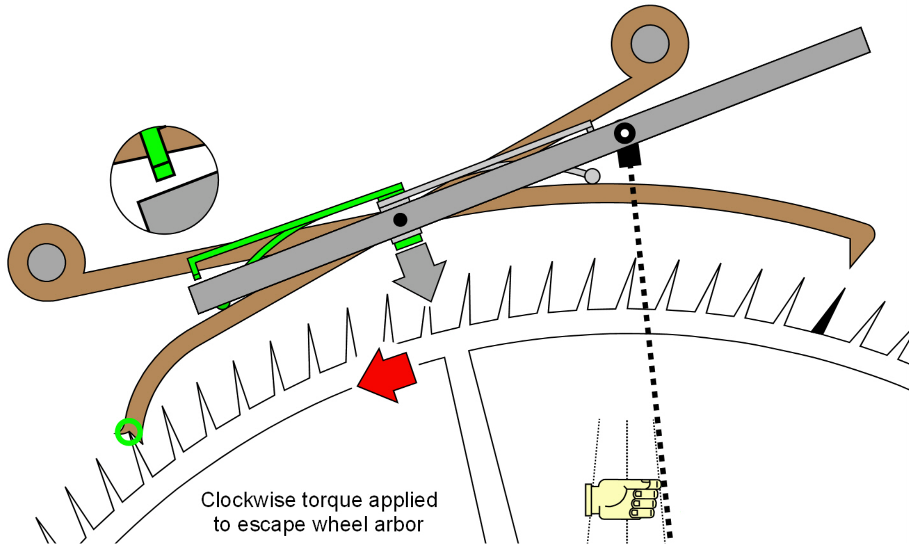


Figure 20 - Pendulum overswing to the right. Escape wheel recoil continues. Entry pallet remains captured. Entry composer lifted further away from escapement frame.

COMPLETE CYCLE OF OPERATION

Figure 21 - This explanation of one complete cycle of operation commences a brief instant after the situation depicted in Figure 20, with all manual inputs removed. Observe the position of the black escape wheel tooth, which will record advancement of the escape wheel during the cycle. Clockwise torque to the pendulum is being generated by the clockwise driven escape wheel, which is transmitting its energy, in sequence, to the captured entry pallet nib locking corner, entry pallet arm, single pallets pivot pin, escapement frame, escapement frame arbor, crutch (not illustrated) and pendulum. Under the combined influences of clockwise escapement torque and the Earth's gravity, the overswung pendulum, rigidly attached escapement frame, single pallets pivot pin and resting exit components are accelerating clockwise, about the escapement frame arbor. The captured entry pallet arm is therefore obliged to rotate anticlockwise about the single pallets pivot pin, whereby the raised (green) entry composer (see magnified inset), which is moving in unison with the entry pallet arm, also rotates anticlockwise about the same pivot, lowering the entry composer stops closer to the escapement frame.

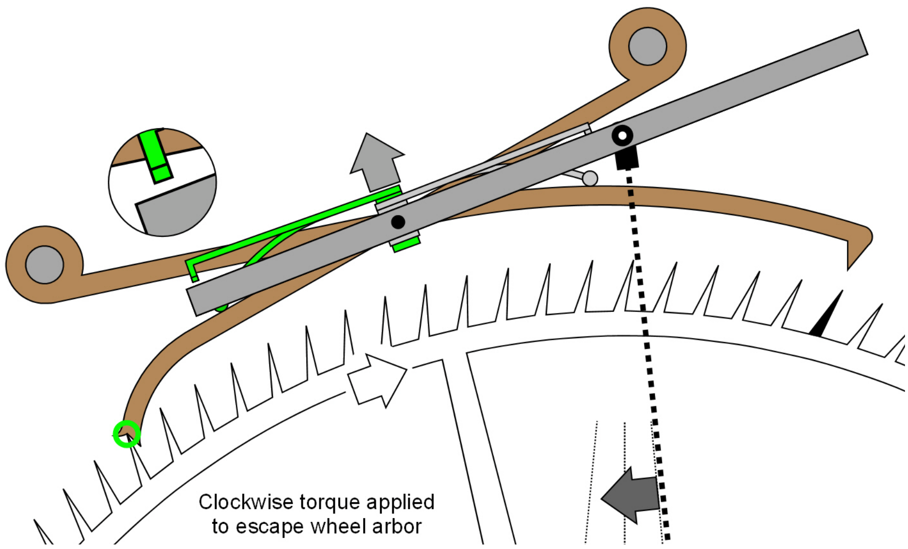


Figure 21 - No manual inputs. Returning from the limit of overswing to the right. Normal (clockwise) escape wheel rotation. Entry pallet nib locking corner impulse assists gravity, accelerating the pendulum from right to left. Raised entry composer stops lowered towards the escapement frame.

Figure 22 - Under the continuing influences of clockwise escapement torque and the Earth's gravity, the pendulum, rigidly attached escapement frame, single pallets pivot pin and resting exit components are still accelerating clockwise about the escapement frame arbor axis. The pendulum is now passing through the right-hand pendulum displacement marker, at the cessation of overswing to the right. At that same instant, the anticlockwise rotations of the entry pallet arm and entry composer about the single pallets pivot pin have just reached the point at which the entry composer stops have contacted the escapement frame, as highlighted by the broken green circle and by the entry composer changing from green to grey. Entry composer arrest is normally gentle and, at most, barely audible.

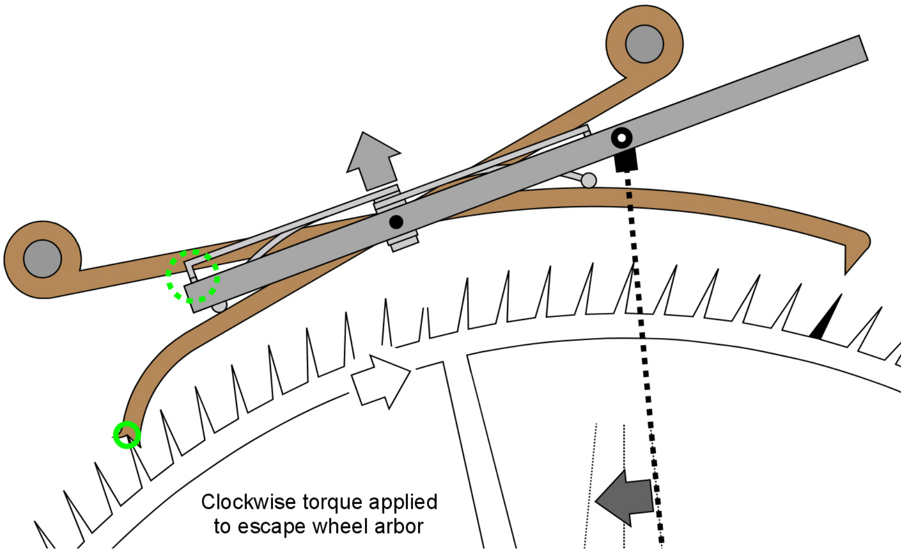


Figure 22 - End of overswing to the right. Entry pallet nib locking corner impulse and gravity maintain pendulum acceleration to the left. Entry composer halted by escapement frame.

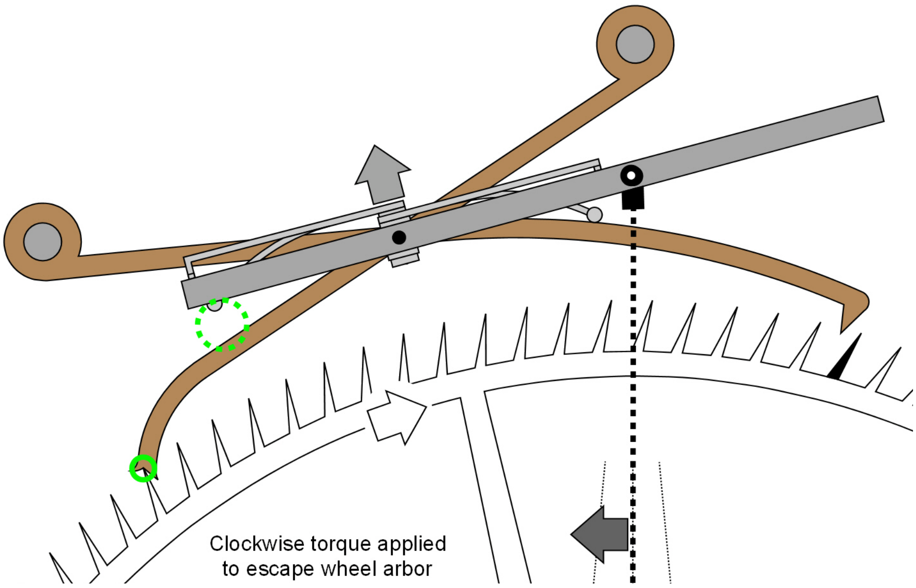


Figure 23 - Entry pallet nib locking corner impulse continues.
Entry pallet arm detached from resting entry composer.

Figure 23 - The pendulum is passing through mid swing. The entry pallet arm has continued to rotate anticlockwise about the single pallets pivot pin, whilst the entry composer has been left behind, resting upon the escapement frame. At the illustrated mid-swing position, the entry pallet arm has separated from the entry composer to an obvious extent, highlighted by the broken green circle. Observe that the exit pallet nib is swinging towards the escape wheel, as the escapement frame rotates clockwise about the escapement frame arbor. From the illustrated position, gravity will begin to oppose escapement impulse and pendulum deceleration must therefore begin.

Figures 24 and Figure 25 - These separate figures represent several events occurring virtually simultaneously, but most clearly described in two separate stages.

Figure 24 - The exit pallet nib has continued to rotate clockwise about the escapement frame arbor, to the extent that it has just encountered the escape wheel. The escapement geometry must be designed such that contact between the exit pallet nib and an appropriate escape wheel tooth tip occurs precisely at the exit nib locking corner at the instant the pendulum coincides with the left-hand displacement marker. Subject to materials used and contact speed, a 'click' sound might be generated. The escape wheel is instantaneously halted and impulse to the entry pallet nib locking corner ceases.

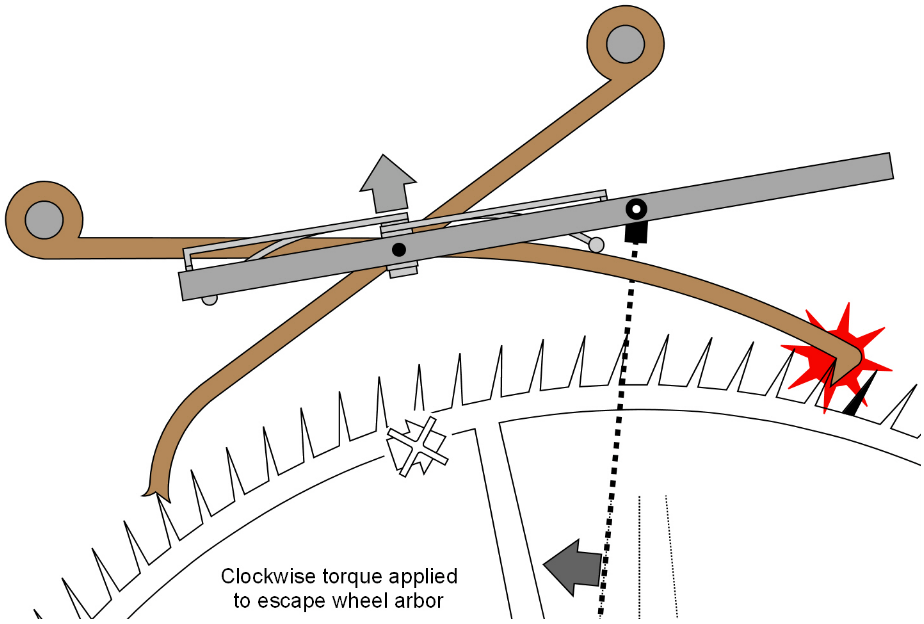


Figure 24 - Pendulum coincides with left-hand displacement marker. Exit pallet nib locking corner contacts escape wheel tooth tip. Escape wheel halted. End of entry pallet nib locking corner impulse.

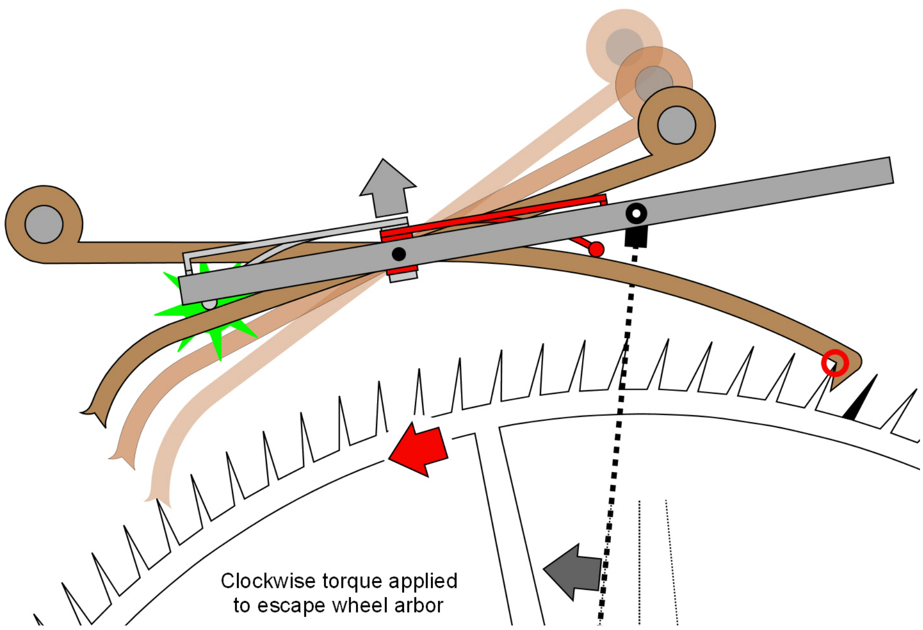


Figure 25 - Virtually coincident with Figure 24. Exit nib locking corner captured. Exit composer raised. Escape wheel recoil begins. Entry pallet arm released and arrested by entry composer. Exit nib locking corner impulse and gravity decelerate the pendulum.

Figure 25 - The pendulum swings further to the left by an imperceptibly small amount. The exit nib locking corner is captured, the escape wheel is recoiled, the entry nib locking corner is thereby released and, after a brief period of free motion, the entry pallet arm is arrested by the entry composer. Subject to entry pallet arm tail weighting and inertia, there may be an audible 'click' as the arm meets the composer. There may be further clicks if there is arm and/or composer bounce. The exit pallet arm and paired composer begin to rotate imperceptibly anticlockwise about the single pallets pivot pin, lifting the exit composer stops away from the escapement frame (exit composer changes from grey to red). Escape wheel impulse applied to the exit pallet nib locking corner generates anticlockwise torque about the escapement frame arbor, assisting gravity in decelerating the pendulum.

Figure 26 - Exit pallet nib locking corner impulse and gravity oppose diminishing pendulum momentum during increasing overswing to the left. Increasing exit composer lift is more clearly shown in the magnified inset. All motions will eventually stop for an instant, at the limit of overswing to the left (not illustrated).

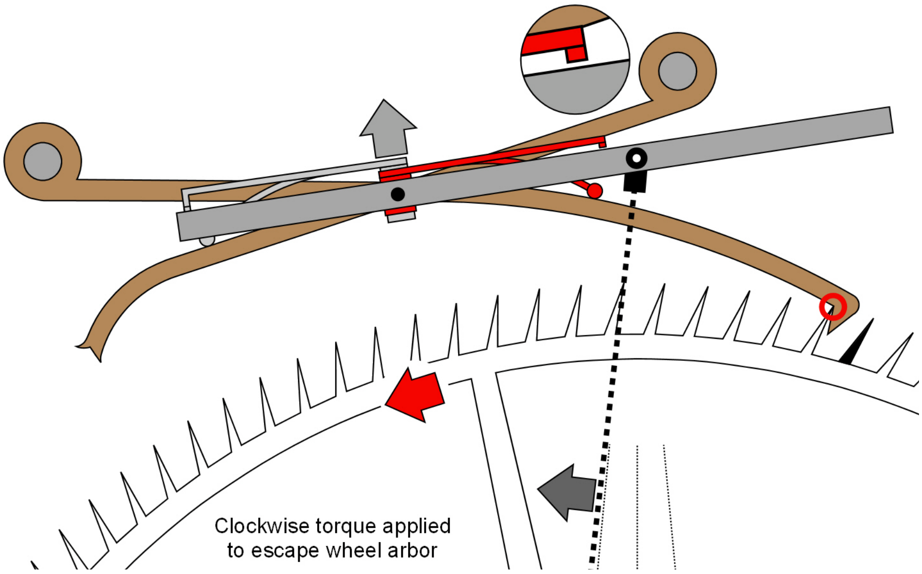


Figure 26 - Continuing overswing to the left and escape wheel recoil. Further exit composer lift.
Exit pallet nib locking corner impulse and gravity continue the deceleration of the pendulum.

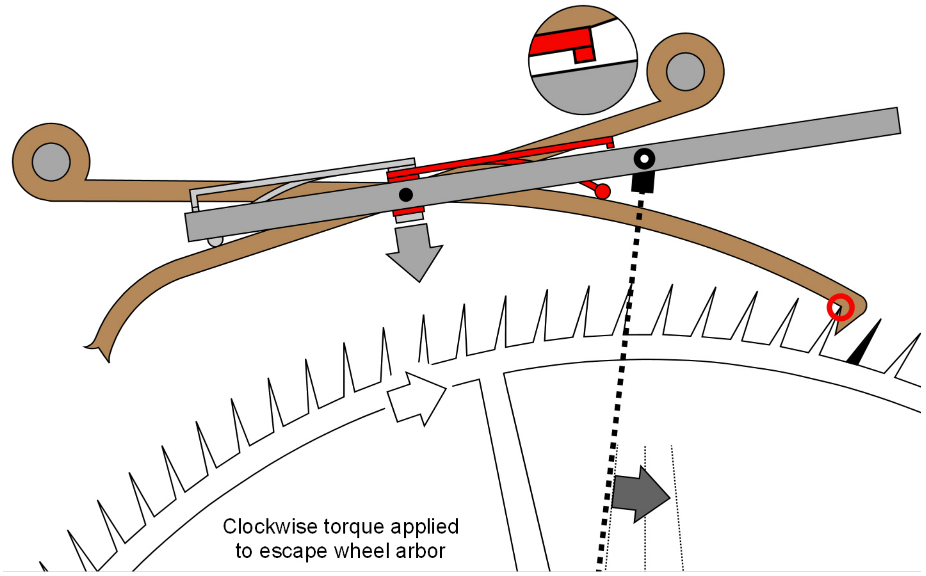


Figure 27 - Return from overswing to the left. Normal, clockwise escape wheel rotation.
Exit pallet nib locking corner impulse assists gravity in accelerating the pendulum from left to right.
Raised exit composer lowered ever closer towards the escapement frame.

Figure 27 - From the point at which overswing to the left is reversed, gravity and exit pallet nib locking corner impulse combine to accelerate the pendulum to the right during decreasing overswing, whilst the rigidly attached escapement frame rotates anticlockwise about the escapement frame arbor. The captured exit pallet arm and the exit composer rotate clockwise about the single pallets pivot pin, lowering the raised exit composer stops ever closer to the escapement frame (see magnified inset)

Figure 28 - Overswing to the left is at an end, as indicated by the left hand pendulum displacement marker. At that same instant, the clockwise rotations of the exit pallet arm and entry composer about the single pallets pivot pin have just reached the point at which the exit composer stops have contacted the escapement frame (exit composer changes from red to grey), as highlighted by the broken red circle. Exit composer arrest should be a gentle and, at most, barely audible event. The exit pallet nib locking corner is still receiving impulse from the escape wheel and is continuing to assist gravity in accelerating the pendulum to the right.

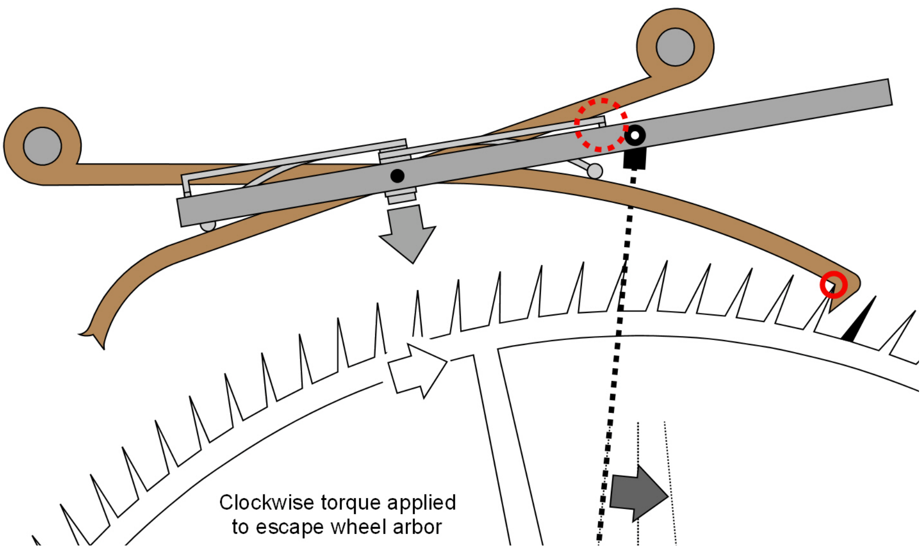


Figure 28 - End of overswing to the left. Exit pallet nib locking corner impulse and gravity maintain pendulum acceleration to the right. Exit composer halted by escapement frame.

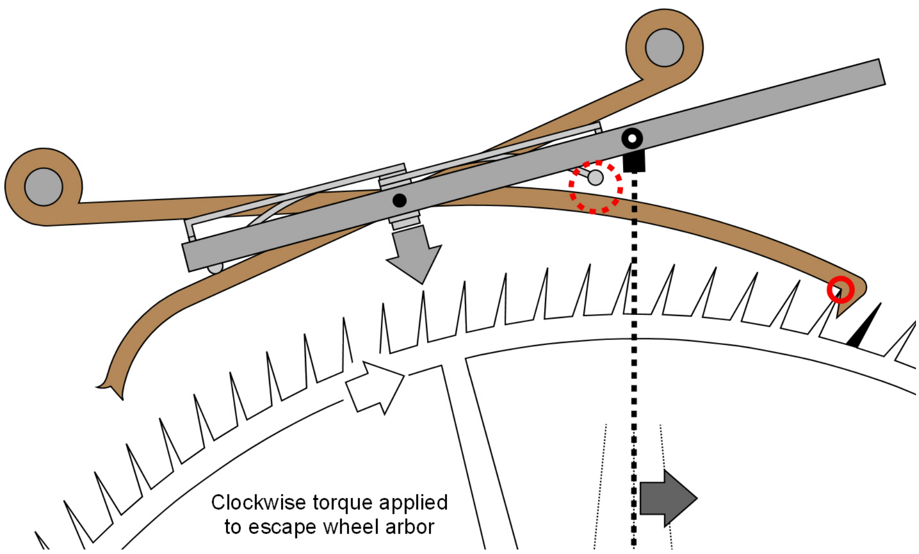


Figure 29 - Exit pallet nib locking corner impulse continues. Exit pallet arm detached from exit composer.

Figure 29 - The exit pallet arm continues to rotate clockwise about the single pallets pivot pin, thereby breaking contact with the exit composer, as highlighted by the broken red circle. The exit composer is left behind, its stops resting upon the escapement frame. As illustrated, the pendulum is passing through mid-swing and the exit pallet arm is well clear of the composer. Observe that the entry pallet nib is swinging towards the escape wheel, as the escapement frame and single pallets pivot pin rotate anticlockwise about the escapement frame arbor. From the illustrated position, gravity will oppose escapement impulse and pendulum deceleration must begin.

Figures 30 and 31 - These separate figures represent events occurring virtually simultaneously.

Figure 30 - The entry pallet nib has continued to rotate anticlockwise about the escapement frame arbor, in unison with the escapement frame, to the extent that it has just encountered the escape wheel. The escapement geometry must be designed such that contact between the entry pallet nib and an appropriate escape wheel tooth tip occurs precisely at the entry nib locking corner at the instant the pendulum coincides with with the right-hand displacement marker. Subject to the materials used and the speed of contact, a 'click' sound might be generated. The escape wheel is instantaneously halted and exit pallet nib locking corner impulse ceases.

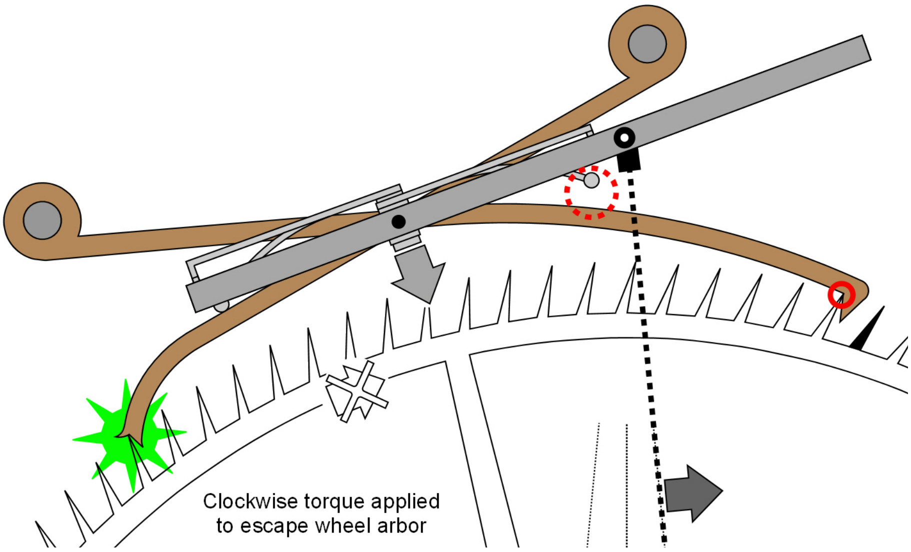


Figure 30 - Pendulum coincides with right-hand displacement marker. Entry pallet nib locking corner contacts escape wheel tooth tip. Escape wheel halted. End of exit pallet nib locking corner impulse.

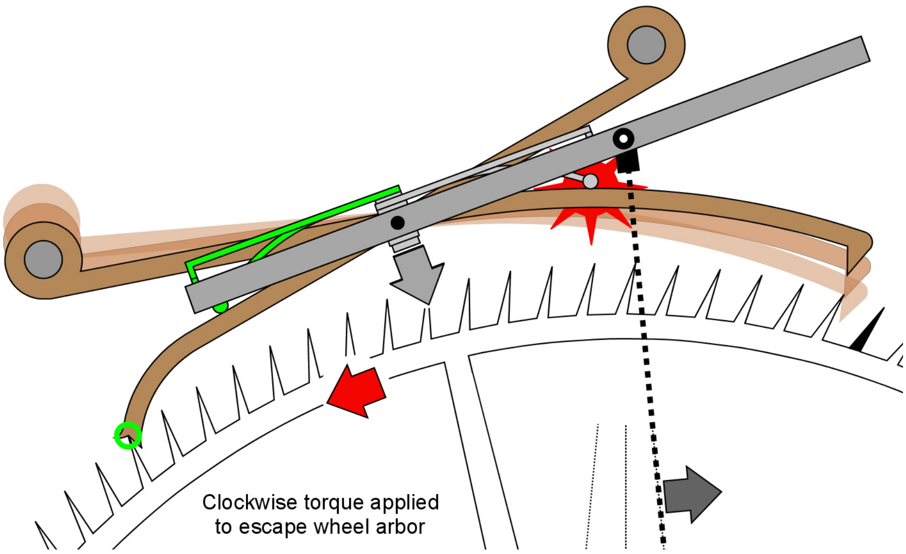


Figure 31 -Virtually coincident with Figure 30. Entry nib locking corner captured. Entry composer raised. Escape wheel recoil begins. Exit pallet arm released and arrested by exit composer. Entry nib locking corner impulse and gravity decelerate the pendulum.

Figure 31 -The pendulum swings further to the right by an imperceptibly small amount. The entry nib locking corner is captured, the escape wheel is recoiled, the exit nib locking corner is thereby released and, after a brief period of free motion, the exit pallet arm is arrested by the exit composer. Subject to exit pallet arm tail weighting and inertia, there may be an audible 'click' as the arm meets the composer. There may be further clicks if there is arm and/or composer bounce. The entry pallet arm and paired composer begin to rotate imperceptibly clockwise about the pallets pivot pin, lifting the entry composer stops away from the escapement frame (entry composer changes from grey to green). Escape wheel impulse applied to the entry pallet nib locking corner generates clockwise torque about the escapement frame arbor, assisting gravity in decelerating the pendulum.

Figure 32 - Entry pallet nib locking corner impulse and gravity oppose diminishing pendulum momentum during increasing overswing to the right. Increasing entry composer lift is more clearly shown in the magnified inset. All motions will eventually stop, for an instant, at the limit of overswing to the right (not illustrated). From that point, the escapement will return to the situation depicted in Figure 21 and the entire sequence will be repeated. Continuous operation is thereby maintained.

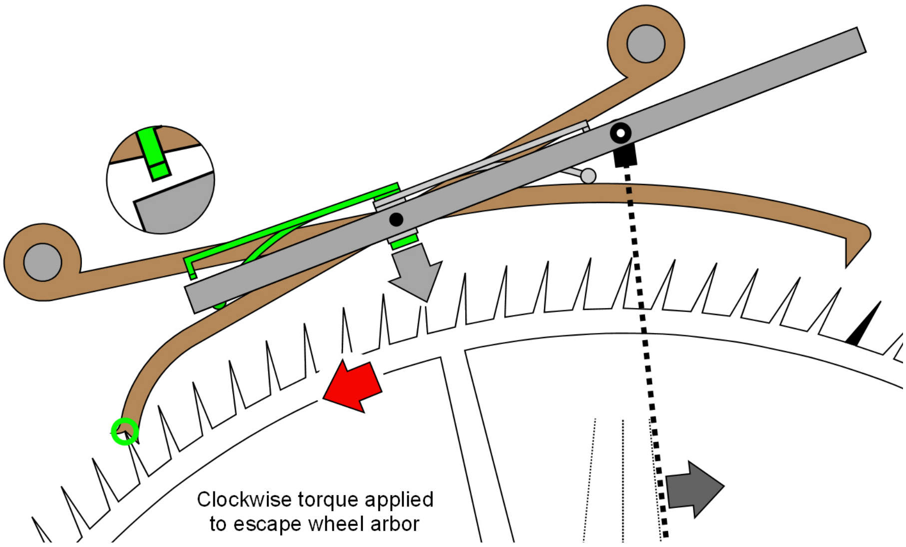


Figure 32 -Continuing overswing to the right and escape wheel recoil. Further entry composer lift. Entry pallet nib locking corner impulse and gravity continue the deceleration of the pendulum.

FOUR MINUTE ESCAPE WHEEL

One complete escapement cycle, occupying two seconds in association with a seconds beating pendulum, has now been described. Observe that, since the black escape wheel tooth has advanced clockwise through one tooth space, the 120 tooth escape wheel would complete one full rotation in 2 x 120 = 240 seconds (four minutes).

NO SLIDING FRICTION, NO WEAR, NO LUBRICATION

A review of the complete cycle of operation will confirm that at no point does sliding friction occur, apart from very limited rotations of the pallet arms and composers about the single pallets pivot pin. There is, therefore, no requirement that the escapement be lubricated. In fact, lubrication of the pallet nibs would reduce essential static friction between captured nib locking corners and capturing escape wheel tooth tips, with potentially ruinous consequences. Furthermore, any lubricant at any of the pivots or nibs, be it modern or ancient, fresh or stale, will suffer unavoidable alterations to its properties with use, exposure too the environment and the passage of time, with potentially significant cumulative effects upon consistent timekeeping.

Negligible sliding friction generates negligible wear and causes negligible changes in performance. Harrison's grasshopper escapement thereby entirely avoids all common escapement problems in a typically direct and thorough fashion, by effectively eliminating their causes at source.

HARRISON'S STIPULATIONS

It is a sad and effectively universal truth that, despite almost two and a half centuries since Harrison's death, no grasshopper escapements have been described, drawn, designed or constructed in strict, simultaneous accordance with every one of his documented stipulations and guidance. Fortunately, detailed analyses of his final, 1775 manuscript and an associated escapement illustration have identified clear instructions. It should, however, be understood that even the most perfect Harrison grasshopper escapement would, in isolation, be incapable of achieving his astonishing claims for timekeeping performance. Harrison's other remarkable inventions must also be correctly understood, designed, constructed, installed and adjusted. Sadly, their explanation is beyond the scope of this publication.

WRITTEN STIPULATIONS

In 1775, the year before his death, Harrison recorded many of his horological principles in a significant manuscript entitled: 'A DESCRIPTION CONCERNING SUCH MECHANISM AS WILL AFFORD A NICE, OR TRUE MENSURATION OF TIME; TOGETHER WITH SOME ACCOUNT OF THE ATTEMPTS FOR THE DISCOVERY OF THE LONGITUDE BY THE MOON: AS ALSO AN ACCOUNT OF THE DISCOVERY OF THE SCALE OF MUSICK. Commonly referred to as **'Concerning Such Mechanism'** or simply **'CSM'**, the manuscript included many (though by no means all) of Harrison's stipulations for the performance of his single pivot grasshopper escapement, interpretations of which are listed below. Stipulation numbering is arbitrary and should neither be attributed to Harrison nor be taken to indicate any degree of importance.

- **STIPULATION 1 - There must be no sliding friction and (therefore) no wear or requirement for lubrication.** This achievement was demonstrated during the earlier explanation of the operating cycle. Variations in friction and resistance due to wear and lubricant degradation can be a significant cause of inconsistent timekeeping performance.

- **STIPULATION 2 - The mean torque arm should be one hundredth of the pendulum length.** Harrison's broad objective is that impulse be applied sensibly close to the point of pendulum suspension, thereby restricting the undesirable influence of the escapement upon the natural, free motions of the pendulum. In addition, he is encouraging higher impulse forces, whereby unavoidable variations become a less significant proportion of the whole. Regrettably, Harrison fails to define his exact intentions for 'pendulum length'. Fortunately, the breadth of Harrison's objective suggests that a precise implementation is not essential. The author considers idealised pendulum length to be the most logical and sensible basis, because it defines a single, universal escapement geometry for all timepieces with the same pendulum period (and, strictly speaking, the same acceleration due to gravity). Nevertheless, in an effort to avoid pointless controversy, the entirety of this publication will merely stipulate a mean torque arm of 10 mm (1/100th of 1 metre) for final geometries. This cannot be far from Harrison's broad intention, whatever it was. Fortunately, the mean torque arm may be altered with ease, if so desired.

- **STIPULATION 3 - The pendulum arc should be large, although fifteen degrees should not be exceeded.** The grasshopper escapement has an invaluable capacity to function as required at extremely large escaping arcs. Harrison imposes a large arc to ensure that unavoidable variations are rendered a small, less influential proportion of the whole. There are additional influences, beyond the scope of this publication, although they have no bearing upon the process of designing compliant geometries.

- **STIPULATION 4 - A 'long pendulum', beating seconds, must be incorporated.** A seconds beating pendulum will be assumed throughout this publication.

- **STIPULATION 5 - An escape wheel rotating once every four minutes shall be used.** As concluded during the description of the escapement cycle, this stipulation requires an escape wheel of one hundred and twenty teeth in combination with the seconds beating pendulum dictated by Stipulation 4.

- **STIPULATION 6 - See 'CONCLUSIONS - STIPULATION SIX' (page 29) for Harrison's instructions relating to the development of impulse during each grasshopper escapement cycle.** CSM is ambiguous and incomplete. Harrison states: *'...let, as I order the Matter, the Force [from the Wheel] upon the Pendulum, as just before the interchanging of the Pallats, to be as by or from them the said Pallats supposed or taken as 3, then as just after their interchanging [and the Force to contrary Direction], it must be about as 2, that is, it must be so ordered [as may hereafter be observed by the Drawing] viz as that it be so by the taking, or supposing for the Purpose, a Mean betwixt the Actions of each Pallat…'*. Sadly, Harrison later declares that *'...the Drawing…'* and a more detailed explanation will no longer be offered, in response to poor treatment and incomplete reward by the Board of Longitude. It is considered fair to conclude that Harrison's deliberate exercise in self-sabotage has thus far committed his remarkable invention to almost two and a half centuries of effectively universal misrepresentation.

ILLUSTRATED STIPULATIONS

The following analysis will support a proposal that *'…the Drawing…'* promised in CSM (see above), but ultimately withheld, was created after all and that it still exists. Figure 33 reproduces a scanned photocopy of the object in question, commonly referred to as 'MS3972/3'. As will be demonstrated, the illustration incorporates a complete and unambiguous definition of Harrison's precise intentions for the delivery of impulse, together with invaluable clues relating to the manipulation of escaping arc.

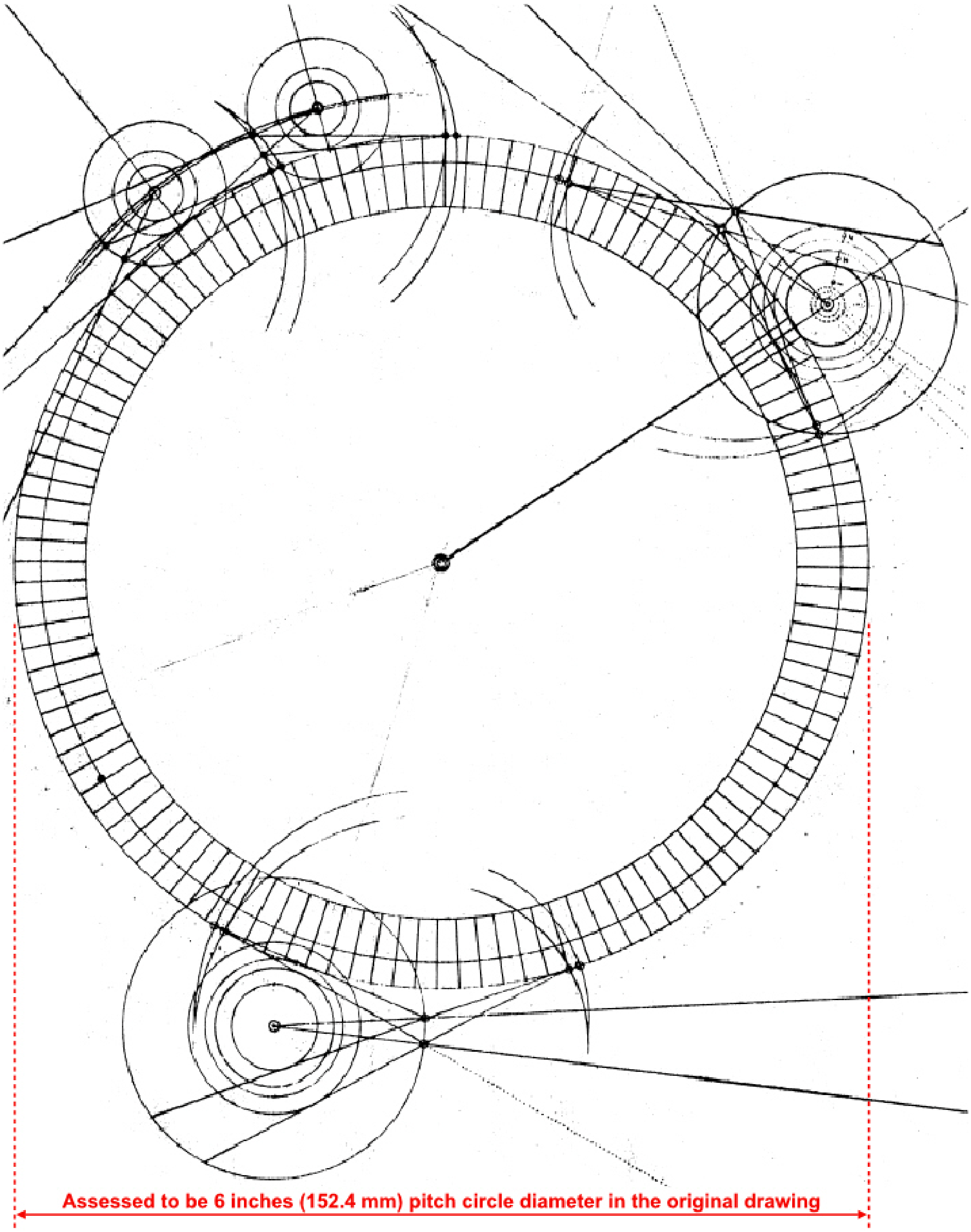


Figure 33 - MS3972/3, not to scale, edges cropped, red lines and text added.

Alterations to MS3972/3 for the purposes of this publication are limited to slight cropping and scaling and an added dimension, in red, assessing the largest, clockwise rotating escape wheel teeth tips pitch circle diameter (PCD) to be six inches. A slightly smaller concentric circle defines the PCD of a second escape wheel. The innermost circle, which may be ignored, is assessed to be five inches in diameter. PCD intersections with one hundred and twenty equally spaced escape wheel radials define the teeth tips of either escape wheel. Encouragingly, CSM also stipulates a 'four-minute' escape wheel (120 teeth in combination with a seconds beating pendulum).

Three complete grasshopper escapement geometries are arranged around the escape wheels as follows:

(1) - The two adjacent constructions to the upper left constitute Harrison's 'longitude' escapement, applicable to twin balances swinging in opposition.

(2) - The lower geometry is a of single pivot configuration, spanning a mean of 16.5 escape wheel tooth spaces (16 tooth spaces minimum, 17 maximum, therefore 16.5 mean) of the larger escape wheel.

(3) - The geometry to the upper right is of a single pivot configuration, spanning a mean of 17.5 escape wheel tooth spaces (17 tooth spaces minimum, 18 maximum, therefore 17.5 mean) of the smaller escape wheel.

Each of the geometries simultaneously represents all start and end of impulse events during one complete cycle. Anticlockwise escape wheel recoil and supplementary arc are not incorporated.

It is considered highly probable that MS3972/3 was an explanatory illustration, not an accurate design drawing. Apart from the wide range of applications, any competent 18th century designer faced with the task of creating accurate escapement geometries would surely have constructed them to a far greater scale and reduced all linear measurements arithmetically, in an effort to improve precision. Regrettably, all available copies of the original illustration have, thus far, exhibited non-linear distortions. Therefore, MS3972/3 measurements, be they from the original or a copy, cannot be entirely trusted. Fortunately, completely regardless of any such deficiencies, it is an inarguable certainty that written numbers are immune, as will be revealed.

ANALYSIS OF THE 17.5 MEAN SPAN MS3972/3 GEOMETRY

Figure 34 reproduces the 17.5 mean tooth spaces spanned single pivot geometry from the upper right of Figure 33, rotated anticlockwise through approximately ninety degrees and greatly enlarged to no particular scale. Author alterations are limited to the addition of circled crosses at eleven points of intersection, Arial font labelling and a block of text, all in red. Normal escape wheel rotation is clockwise.

THE ANALYSIS AND RELEVANT GUIDANCE IS PRESENTED IN FIVE PARTS:

PART ONE - PALLET ARMS LINES OF ACTION

Figure 34 includes four pallet arms lines of action, which define the directions of the start and end of impulse forces generated by supplied escape wheel energy during one complete cycle. Note that the magnitudes of those forces are not represented. The escapement frame arbor (crutch arbor) axis is labelled as 'Z'. Relevant properties are as follows:

(1) - CJ represents the entry pallet arm line of action at the start of entry pallet impulse. The compressive force, acting from J to C (extended) at perpendicular torque arm LZ, will apply clockwise torque about Z.

(2) - DK represents the entry pallet arm line of action at the end of entry pallet impulse. The compressive force, acting from K to D (extended) at perpendicular torque arm MZ will apply clockwise torque about Z.

(3) - AD represents the exit pallet arm line of action at the start of exit pallet impulse. The tensile force, acting from D to A at perpendicular torque arm FZ, will apply anticlockwise torque about Z.

(4) - BC represents the exit pallet arm line of action at the end of exit pallet impulse. The tensile force, acting from C to B at perpendicular torque arm GZ, will apply anticlockwise torque about Z.

Ignoring recoil (irrelevant to this analysis), adequate torques, applied in a repeating sequence (1) - (2) - (3) - (4), would maintain the continuous motion of a pendulum.

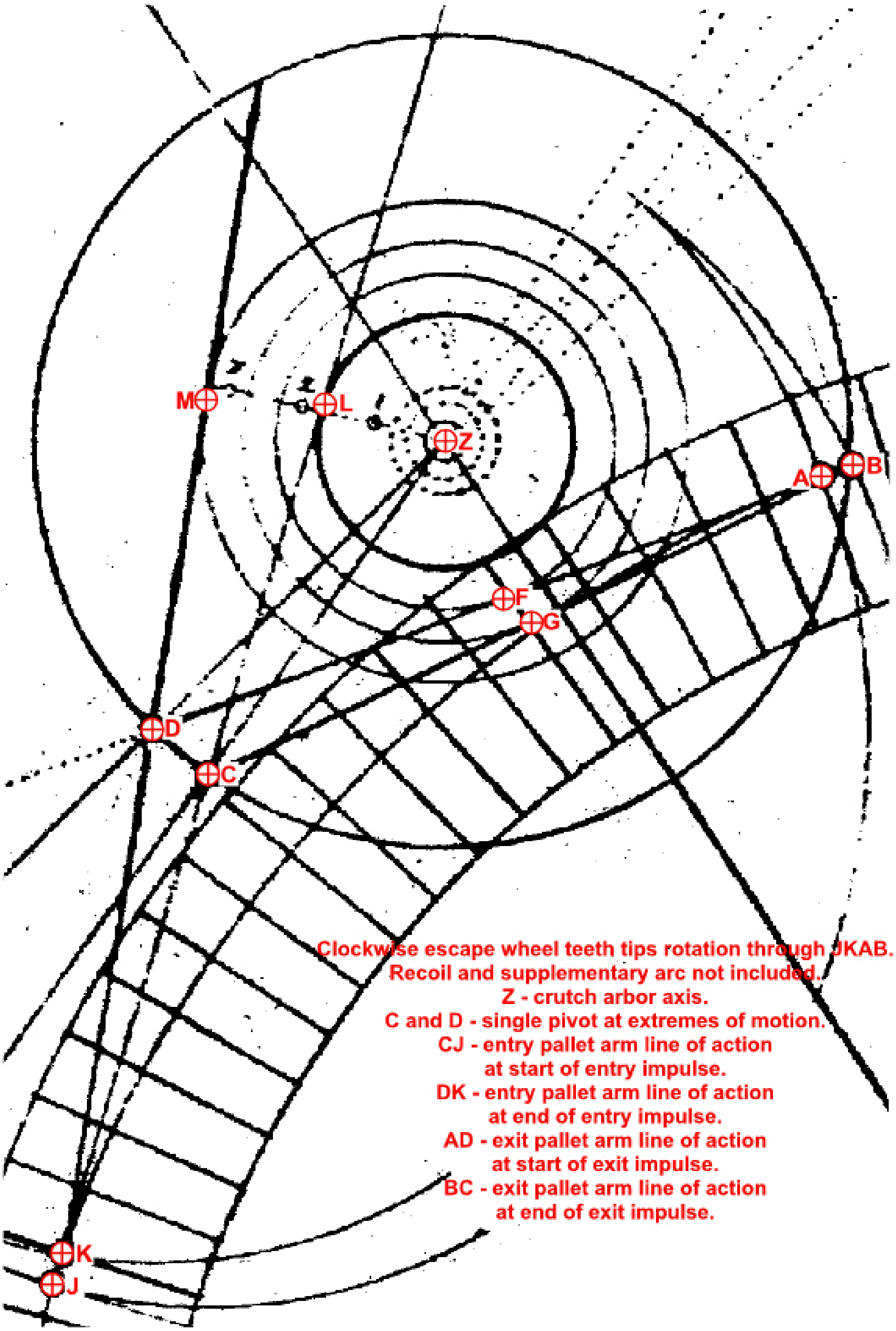


Figure 34 - MS3972/3 17.5 mean tooth spaces spanned, single pivot geometry illustration.

PART TWO - HARRISON'S THREE NUMBERED POINTS

Harrison has inserted three separate single-digit numbers '1', '2' and '3', arranged in that order. Each of those numbers is associated with a very small circle, all three of which lie along a common radial from the escapement frame arbor (crutch arbor) axis, Z. Very small circles such as these are a drawing convention consistently used by Harrison to define the location of points, deemed to be located 'invisibly' at their centres. The three Harrison numbers are, therefore, each associated with defined points along the common radial.

With due regard for the deficiencies of the original or copied illustration, it is proposed that the adjacent separations of the points labelled Z, '1', '2', and '3' were intended to be equal. On that basis, if the distance from the point labelled 'Z' to the adjacent point labelled '1' is called 'd', then:

Distance Z to point '2' = 2d…………...(1)

Distance Z to point '3' = 3d…………...(2)

PART THREE - TORQUE ARM CIRCLES

Harrison includes four concentric circles, referred to herein as 'torque arm circles', all centred at Z. The radii of the torque arm circles match the torque arms LZ, MZ, FZ and GZ, derived earlier. Their purpose is to illustrate the transference of the lengths of all four torque arms to Harrison's common radial from Z.

With due regard for the deficiencies of the original or copied illustration, it is proposed that the centre of small circle '2' is equidistant from the two inner torque arm circles, of radii LZ and FZ, which represent the entry and exit start of impulse torque arms for one complete escapement cycle of operation. It is also proposed that the centre of small circle '3' is equidistant from the two outer torque arm circles, of radii MZ and GZ, which represent the entry and exit end of impulse torque arms for one complete cycle.

Therefore: Distance Z to point '2' = 0.5 (LZ + FZ)…………...(3)

and: Distance Z to point '3' = 0.5 (MZ + GZ)………….(4)

From (3) and (1): 0.5 (LZ + FZ) = 2d……………………….(5)

From (4) and (2): 0.5 (MZ + GZ) = 3d……………………….(6)

Therefore, from (5) and (6): (MZ + GZ) / (LZ + FZ) = 3 / 2……………………. (7)

PART FOUR - VARYING FORCES

The geometries of MS3972/3 are obviously incapable of representing the magnitudes of transmitted forces. Constant, equal forces are imposed, whereby torque is rendered equivalent to torque *arm*. In reality, forces within the grasshopper escapement are most certainly not constant (with the assumed exception, for the purposes of this analysis, of escape wheel delivery). An inspection of Figure 2 will confirm that, as a consequence of variations in the orientations of engaged pallet arms lines of action relative to the escape wheel, the transmitted components of force from the escape wheel inevitably alter during each complete cycle. The greater the divergence of any pallet arm line of action from tangential to the escape wheel, the greater will be the reduction in the transmitted component of *'…the Force [from the Wheel] upon the Pendulum…'*, as Harrison expresses it. Of academic interest, the force varies in proportion to the sine of the angle between the line of action and the corresponding escape wheel radial. For a geometry complying with equation (7), above, the practical consequence of such variations is that the ratio of mean end of impulse torque to mean start of impulse torque will differ from the 3 to 2 ratio of mean torque *arms* defined by that equation.

CONCLUSIONS - STIPULATION SIX

From equation (7), above, Harrison's stipulation, ingeniously incorporated within the MS3972/3, 17.5 mean span geometry, is that the ratio of the mean of the end of impulse torque arms to the mean of the start of impulse torque arms (the 'mean end/start ratio', T) should be exactly 3 to 2. This is a precise and unambiguous interpretation of the CSM statement *'…a Mean betwixt the Actions of each Pallat…'*.

CSM includes Harrison's honest acknowledgement of the influence of varying forces. By stipulating a ratio of 3 to *'about as 2'* for '*…the Force [from the Wheel] upon the Pendulum…'*, Harrison is accepting that there are differences between his drawn geometries and escapements constructed in accordance with them. In none of his known manuscripts or illustrations does Harrison propose that any allowances for varying forces be made. He was apparently content to design his escapement on the basis of forces assumed to be constant, although his other CSM and MS3972/3 stipulations and guidance might also have been essential conditions.

MS3972/3 ESCAPING ARC

With regard for the identified deficiencies of the original or copied MS3972/3 single pivot escapement illustrations, both geometries incorporate an escaping arc at or very close to 9.75 degrees.

MS3972/3 LINES OF ACTION

With continuing regard for the identified deficiencies of both MS3972/3 single pivot escapement illustrations, it is considered likely that all entry pallet arm lines of action at the start of impulse were intended to be tangential to the escape wheel PCD. Both single pivot geometries independently support that conclusion. It is, however, possible that the exit lines of action at the start of impulse deviate slightly from tangential in both geometries. It is considered especially relevant that Harrison highlighted long extensions to the exit lines of action at the start of exit impulse in both geometries, painstakingly drawing each as a series of closely spaced dots. It is, surely, unlikely that such emphasis and time consuming drawing effort would have been made and expended to no purpose. Sure enough, thorough and precise analysis has confirmed that very slight alterations to the orientation of the exit pallet arm line of action at the start of exit impulse can have a significant effect upon the geometry. A detailed explanation of the immense value of this observation will be offered towards the end of the design sequence.

COMPUTER AIDED DESIGN OF THE SINGLE PIVOT GEOMETRY

The devised design technique is exceptionally well suited to the task of creating grasshopper escapement geometries. Detailed instructions, explanations and illustrations enable the creation and verification of entire single pivot grasshopper escapement geometries in simultaneous compliance with every one of Harrison's currently identified stipulations, or any chosen variations. Of particular value is a unique capacity to manipulate escaping arc whilst simultaneously maintaining any chosen mean end/start torque arm ratio. By virtue of that unequalled combination and superior flexibility, less capable mathematical methods are rendered entirely obsolete.

COMPUTER HARDWARE AND CAD SOFTWARE

The drawing sequence, explained shortly, originated in 2003 and was expanded in 2011. Precision was assured by inexpensive IMSI TurboCAD Deluxe 15 Computer Aided Design (CAD) software, nevertheless capable of claimed resolutions to ten decimal places. Although TurboCAD is slightly inconsistent at the higher decimal paces, no meaningful consequences arise. The software includes a 'Help' function and tutorials, enabling advancement from CAD novice to the necessary level of competence within a few hours. A seven year old AMD Sempron 3000+ computer and twelve year old Windows 2000 Pro operating system completed all tasks at acceptable speeds. Expensive hardware and software is clearly not essential. The most significant TurboCad 15 deficiency is an inclination to distort and/or displace curves, circles, text and small circular points during transfer to *publishing* software. Despite extensive corrective effort, minor remnants remain.

DRAWING SEQUENCE OVERVIEW

When stripped of the quite extensive explanations and observations, the fundamental design sequence is short, straightforward and with a little practice, speedy.

STEPS ONE to FOUR of the drawing sequence follow a similar path, with additional constructions, to graphical methods described by this author in 2009 and by Peter Hastings in 2011. The former method controlled impulse characteristics *or* escaping arc, whilst the latter only controlled escaping arc and incorporated an unrecognised deviation from truth. All of those deficiencies have been eliminated for the devised design sequence.

STEPS FIVE to TWELVE and subsequent constructions are entirely new and have been devised with the significant objective of *simultaneously* incorporating any chosen mean end/start ratio and any chosen escaping arc. All remaining variables may also be simultaneously controlled. As deduced earlier, Harrison stipulates a mean end/start ratio of torque arms of precisely 3 to 2 for the geometry and that constraint must, without argument, be incorporated if the escapement is to qualify as a true Harrison single pivot grasshopper. Near the end of the design process, the incorporation of a specific mean torque arm requires nothing more than the adjustment of all linear dimension to a common scale. The subsequent determination of instantaneous (zero pallet arm transit time) entry and exit pallet nib lifts after release is extremely straightforward. Exit pallet nib lift can be an influential design consideration, as will be explained.

DESIGNER CHOICES

In preparation for the design sequence, the following must be chosen:

• **The CAD resoultion.** Subject to the software in use, minimum CAD linear and angular resolutions may require selection by the user. Maximum resolution is suggested. Ten decimal places was used.
• **The total number of escape wheel teeth.** Harrison's CSM and MS3972/3 both favour 120 teeth, which was assumed for the example.
• **The mean tooth spaces spanned by the entry and exit pallet nib locking corners during each complete escapement cycle.** An escapement spanning a mean of 17.5 tooth spaces (17 minimum, 18 maximum, therefore 17.5 mean) was chosen
• **The angle between the entry pallet arm line of action at the start of entry impulse and the applicable escape wheel radial at the start of entry impulse.** 90 degrees (tangential to the escape wheel) was chosen, although the devised CAD technique will accomodate any designer choices.
• **The *initial* angle between the exit start of impulse pallet arm line of action and the applicable escape wheel radial at the start of exit impulse.** 90 degrees was chosen, although the devised CAD technique will accomodate any designer choices. The chosen angle will be altered very slightly during the CAD sequence, although the original designer choice may be restored if considered necessary, as will be explained.
• **The target mean end/start (torque arm) ratio, 'T' for the geometry.** Harrison's 3 to 2 ratio was assumed, complying with his MS3972/3 instruction, determined earlier. A later explanation will reveal how any designer-chosen meand end/start ratio may be incorporated.
• **The mean torque arm, 'M'.** A target of 10 mm was assumed. Regardless of all possible interpretations of Harrison's CSM stipulation for mean torque arm, 10 mm must be close to his intention. In operational terms and for the purposes of the CAD sequence, the choice is by no means critical. Subsequent alteration requires nothing more than an appropriate scaling of all linear dimensions, easily performed using CAD.
• **The escaping arc resoultion.** The designer should decide the number of decimal places to which escaping arc must be determined. Although somewhat academic, the example determined escaping arc to four decimal places.
• **The escaping arc.** For a pendulum centre of motion coincident with the escapement frame arbor axis, the pendulum arc is equal to the escaping arc. For this example, an arc of 9.7500 degrees (to the previously chosen four decimal places) was the target, matching the assessed MS3972/3 arc for both single pivot geometries.

DRAWING CONVENTIONS AND SUGGESTIONS

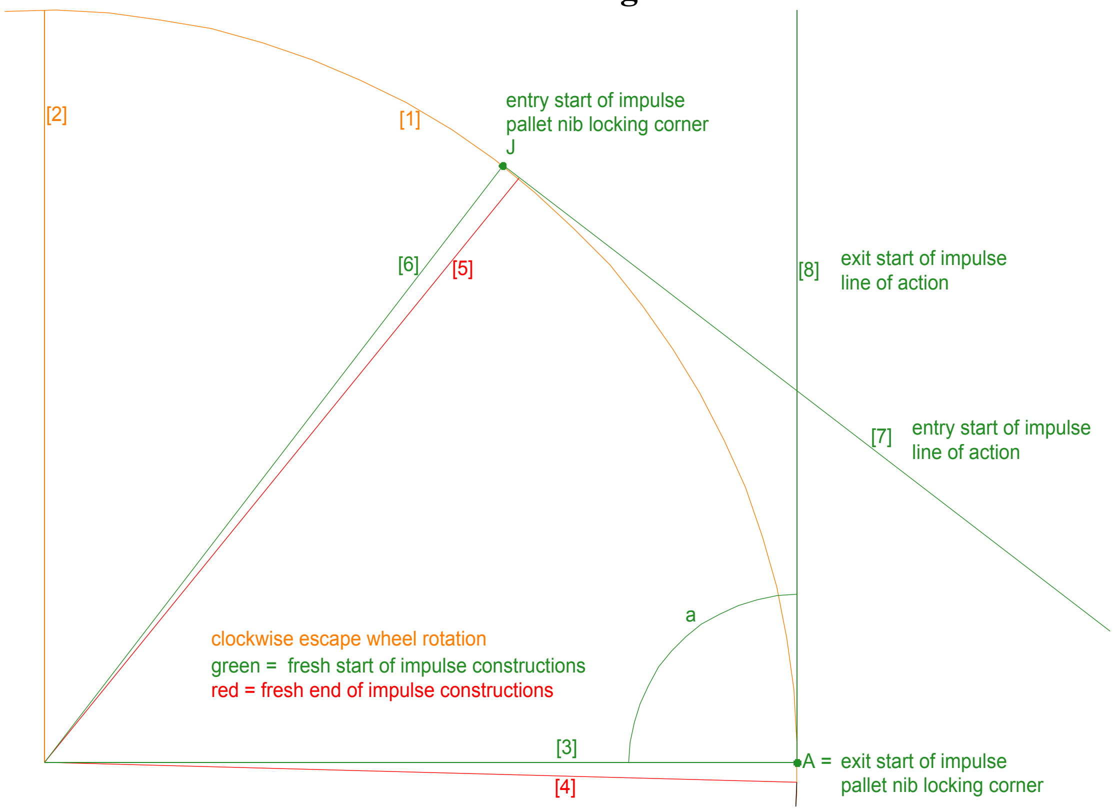
- Numbers thus [NN] in the text refer to identical numbers [NN] in the illustrations.
- **Essential CAD instructions are in bold font**. All remaining text is less essential amplification or explanation.
- Experience suggests that more straightforward CAD construction and fewer mistakes should result if the geometry is drawn 'lying on its side', rather than upright. All illustrations herein are drawn thus. When all CAD construction has been completed, the geometry could, if considered necessary, be rotated as a whole about the escape wheel axis until the crutch arbor was vertically above that axis, as it would in a movement train.
- **Green indicates <u>start</u> of impulse constructions** and **red indicates <u>end</u> of impulse constructions,** unless impracticable. Take care to avoid any confusion with earlier, completely unrelated colour conventions, intended to highlight illustrated entry and exit composer departures from resting positions.
- Unless unsuitable or impracticable, new constructions will be in colour. All prior constructions will be in black or grey**,** including previous start and end of impulse constructions. **Blue may be abitrarily used for emphasis.**
- Normal escape wheel rotation is at all times assumed to be clockwise.
- Recoil and supplementary arc are irrelevant to the creation of the geometry and are not included.
- All CAD drawings should be regarded as purely illustrative and not to scale. As warned earlier, transfer of CAD drawings to *publishing* software may have introduced unavoidable software-induced distortions to and/or slight displacements of curves, circles, text and small circular points.
- All dimensions are millimetres or degrees.

During the CAD design sequence, it will be of considerable value to bear in mind that (ignoring supplementary arc) the single pallets pivot pin has two extremes of motion, at locations C and D, as shown in Figure 34.

Point C corresponds to the *start* of *entry* impulse and the *end* of *exit* impulse.

Point D corresponds to the *end* of *entry* impulse and the *start* of *exit* impulse.

STEP ONE - Figure 35



■ **[1] - Construct a circular arc (shown in orange), of a completely arbitrary radius and of orientation and approximate extent as shown.** Arc [1] is part of the escape wheel pitch circle, along which all escape wheel teeth tips must travel. A radius of 100 mm was chosen for this example.

■ **[2] - Construct a vertical line (shown in orange) upwards from the centre of arc [1], intersecting the arc.** This line, in combination with the next construction, will serve to more clearly define the arc centre.

■ **[3] - Construct a line (shown in green) starting from the centre of arc [1], extending horizontally to the right until it intersects the arc.** The intersection of the line with the arc is labelled 'A'. At the start of exit impulse, the exit pallet nib locking corner is captured by an escape wheel tooth tip located at point 'A'.

■ **[4] - Construct a radial (shown in red) from the centre of arc [1], half an escape wheel tooth space clockwise from line [3].** In this example the chosen 120 escape wheel teeth are spaced 360 ÷ 120 = 3 degrees apart. Half a tooth space is therefore 1.5 degrees. The purpose of this construction will become clear in STEP TWO.

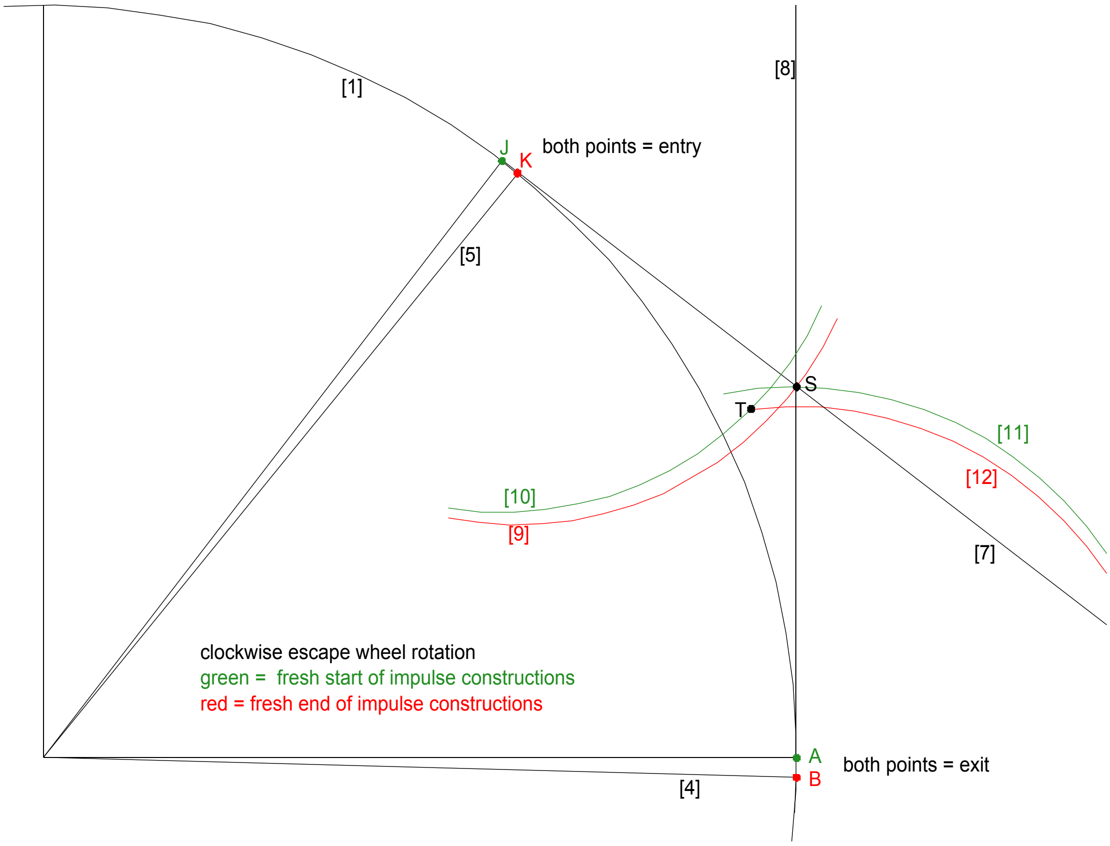
■ **[5] - Construct a radial (shown in red) from the the centre of arc [1], anticlockwise from line [3] by an angle equivalent to the minimum pallet nibs locking corners span.** In this example the chosen 17 tooth spaces minimum span is equivalent to 17 x 3 = 51 degrees between radials [3] and [5]. The purpose of this construction will become clear in STEP TWO.

■ **[6] - Construct a radial (shown in green) from the the centre of arc [1], anticlockwise from <u>radial [4]</u> by an angle equivalent to the maximum pallet nibs locking corners span. NB - The maximum span must be one tooth space greater than the minimum span.** In this example the chosen 18 tooth spaces maximum span defines 18 x 3 = 54 degrees between radials [4] and [6]. The intersection of line [6] with arc [1] is labelled 'J'. At the start of entry impulse, the entry pallet nib locking corner is captured by an escape wheel tooth tip located at point 'J'.

■ **[7] - Construct a (green) line from point 'J' at the designer-chosen angle to radial [6].** In this example, the chosen angle is 90 degrees. Line [7] defines the direction of applied force at the start of entry impulse.

■ **(8) - Construct a (green) line from point 'A' at the designer-chosen angle 'a' to line [3].** In this example, the chosen angle (labelled 'a' for future reference) is 90 degrees. Line [8] defines the initial direction of applied force at the start of exit impulse. This angle will be modified slightly in a later step, although the designer's original choice may be restored if desired, as will be explained.

STEP TWO - Figure 36



The intersection of arc [1] and radial [4] is labelled 'B'.
The intersection of arc [1] and radial [5] is labelled 'K'.
The intersection of line [7] and line [8] is labelled 'S'.

For the remainder of the CAD sequence, any extensions of illustrated arcs or lines will possess the described properties of the illustrated arc or line.

■ **[9] - Construct a circular arc (shown in red), centred at K, passing through S.** Point K is the position of the entry pallet nib locking corner at the end of entry impulse. The radius of arc [9] is a first,very rough estimate of the active length of the entry pallet arm (i.e. the distance between the single pallets pivot pin and the entry nib locking corner). Potential pivot pin locations of the estimated entry pallet arm at the end of entry impulse lie along arc [9].

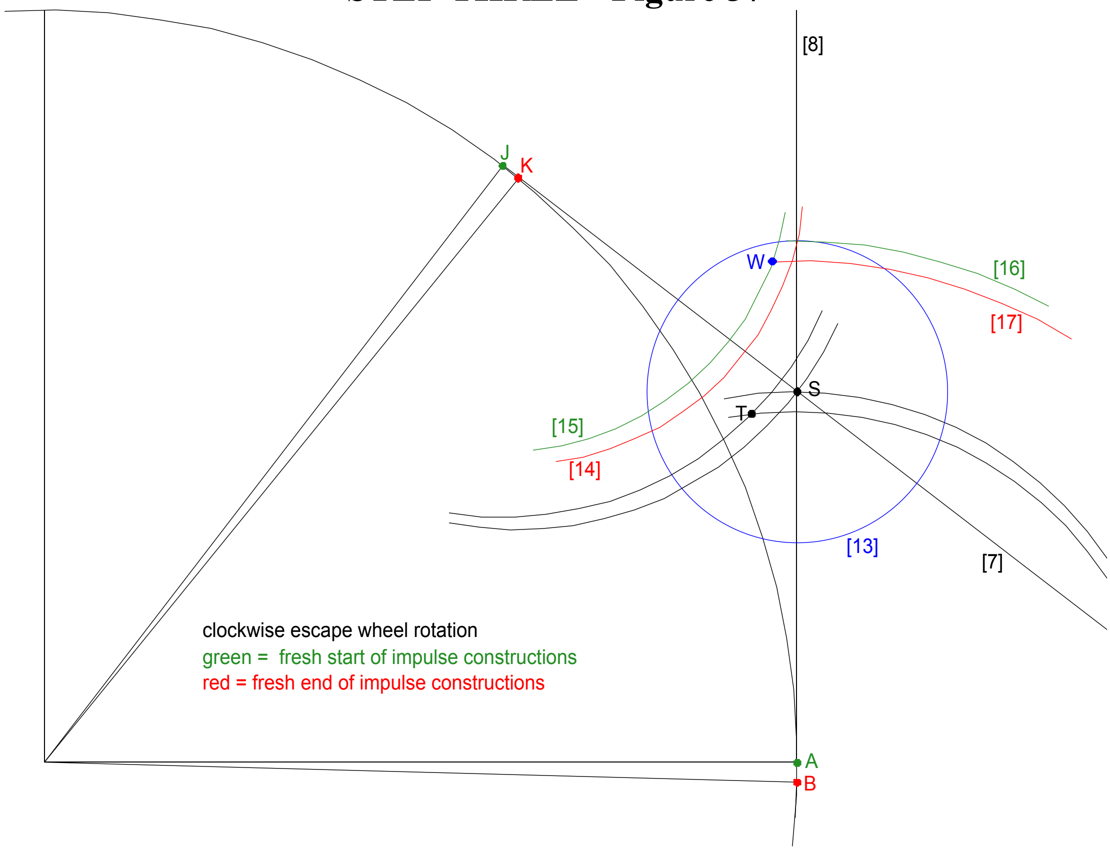
■ **[10] - Construct a circular arc (shown in green), centred at J, of the same radius as arc [9].** Recall from STEP ONE that point J is the position of the entry pallet nib locking corner at the start of entry impulse. Potential pivot pin locations of the estimated entry pallet arm at the start of entry impulse lie along arc [10].

■ **[11] - Construct a circular arc (shown in green), centred at A, passing through S.** Recall from STEP ONE that point A is the position of the exit pallet nib locking corner at the start of exit impulse. The radius of arc [11] is a first, very rough estimate of the active length of the exit pallet arm. Potential pivot pin locations of the estimated exit pallet arm at the start of exit impulse lie along arc [11].

■ **[12] - Construct a circular arc (shown in red), centred at B, of the same radius as arc [11].** Point B is the position of the exit pallet nib locking corner at the end of exit impulse. Potential pivot pin locations of the estimated exit pallet arm at the end of exit impulse lie along arc [12].

The intersection of green start of entry impulse arc [10] and red end of exit impulse arc [12] is labelled 'T'. For the first, very rough estimates of active entry and exit pallet arm lengths, T is a first estimate of the single pallets pivot pin location corresponding to the start of entry impulse and the *simultaneous* end of exit impulse. Unsurprisingly, point T fails to coincide with line [7], the designer-chosen pallet arm line of action at the start of entry impulse. This should be of no concern, as will become apparent in subsequent steps.

STEP THREE - Figure 37



It may be useful to note that, apart from additional construction [13], this step repeats STEP TWO, with the significant exception that it is based upon different rough estimates of entry and exit pallet arms active lengths.

■ **[13] - Construct a circle (shown in blue), centred at point 'S'. A recommended radius is 0.2 of the escape wheel teeth tips pitch circle radius.** In this example, therefore, the radius of [13] is 0.2 x 100 = 20 mm. Circle [13] has been devised as a straightforward means of imposing consistency and establishing a necessary relationship, as will be explained and become clear during the remainder of the CAD design sequence. Therefore, regardless of the initial choice of the radius of circle [13], it is vital that the same radius be maintained for the entire CAD sequence. Should an unsatisfactory choice of [13] become apparent at any time, a revised radius must be applied to a repetition of the CAD sequence from a fresh construction of [13] onwards.

■ **[14] - Construct a circular arc (shown in red), centred at K, passing through the upper intersection of [8] and [13].** The radius of this arc is a second, very rough estimate of the active length of the entry pallet arm at the end of entry impulse. Potential pivot pin locations of the estimated entry pallet arm at the end of entry impulse lie along arc [14].

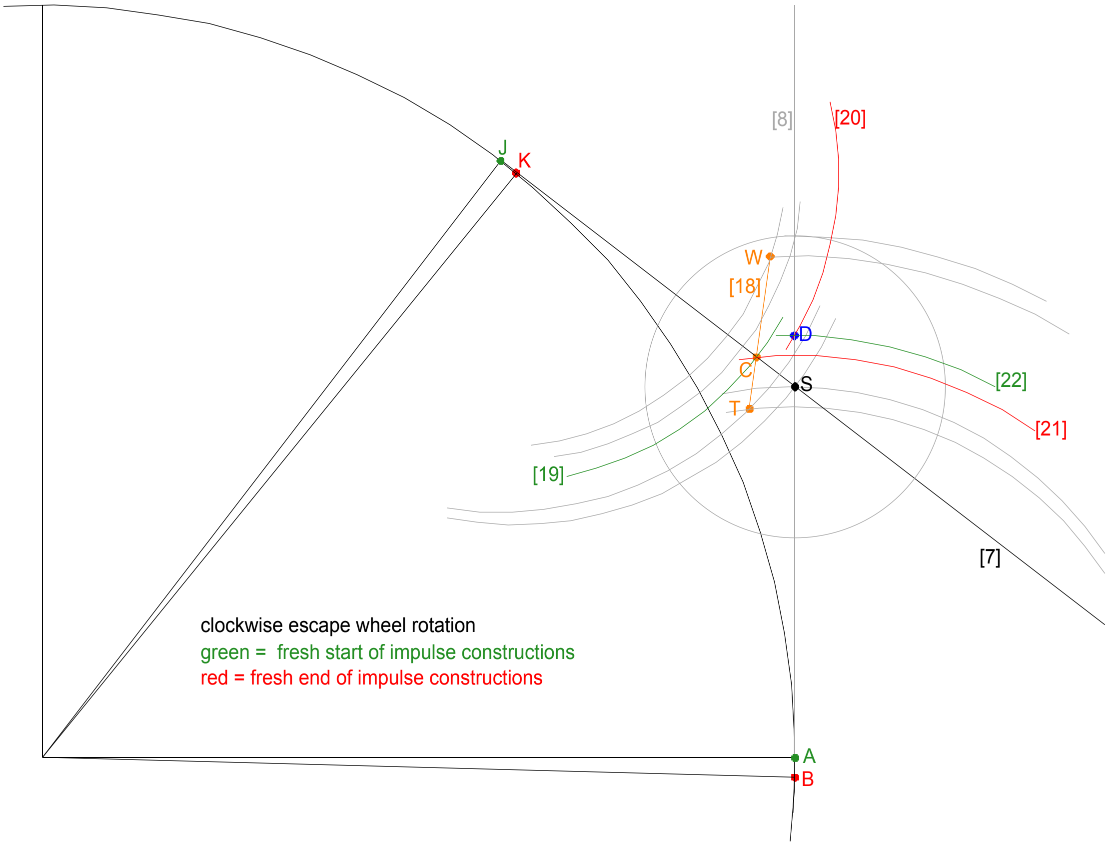
■ **[15] - Construct a circular arc (shown in green), centred at J, of the same radius as arc [14].** Potential entry pallet arm pivot pin positions at the start of entry impulse lie along arc [15].

■ **[16] - Construct a circular arc (shown in green), centred at A, passing through the upper intersection of [8] and [13].** The radius of this arc is a second, very rough estimate of the active length of the exit pallet arm at the start of exit impulse. Potential pivot pin locations of the estimated exit pallet arm at the start of exit impulse lie along arc [16].

■ **[17] - Construct a circular arc (shown in red), centred at B, of the same radius as arc [16].** Potential exit pallet arm pivot pin positions at the end of exit impulse lie along arc [17].

The intersection of green start of entry impulse arc [15] and red end of exit impulse arc [17] is labelled 'W'. For the second, very rough estimates of active entry and exit pallet arm lengths, W is a second estimate of the single pallets pivot pin location corresponding to the start of entry impulse and the *simultaneous* end of exit impulse. Unsurprisingly, point W fails to coincide with line [7], the designer-chosen pallet arm line of action at the start of entry impulse. This should be of no concern, as will become apparent in subsequent steps.

STEP FOUR - Figure 38



■ **[18] - Construct a straight line (shown in orange) between T and W. Extend TW, if necessary, such that it intersects line [7].** The intersection of [7] and [18] is labelled 'C'. Point C will be the final location of the single pallets pivot pin at the start of entry impulse and the simultaneous end of exit impulse (corresponding to point C in figure 34). However, there has been an incorrect assumption that C lies along the same straight line as T and W, whereas it should lie somewhere along a very gentle curve shared by T and W, as may be confirmed by repeating STEPS TWO and THREE for additional rough estimates of the pallet arms active lengths. Rather than attempt to correct the position of point C, which can be extremely tedious, a far more straightforward adjustment will instead be made to the designer-chosen exit pallet arm line of action at the start of exit impulse, as will be explained in STEP FIVE. With that objective in mind, the next four constructions will determine the final location of point D (corresponding to point D in figure 34), based upon to the already established final location of point C. It may be useful to note that the constructions 'backtrack' the constructions of STEP TWO and STEP THREE, by following a sequence: start entry, end entry, end exit, start exit instead of end entry, start entry, start exit, end exit.

■ **[19] - Construct a circular arc (shown in green), centred at J, passing through point C.** The radius of this arc is the final active length of the entry pallet arm at the start of entry impulse.

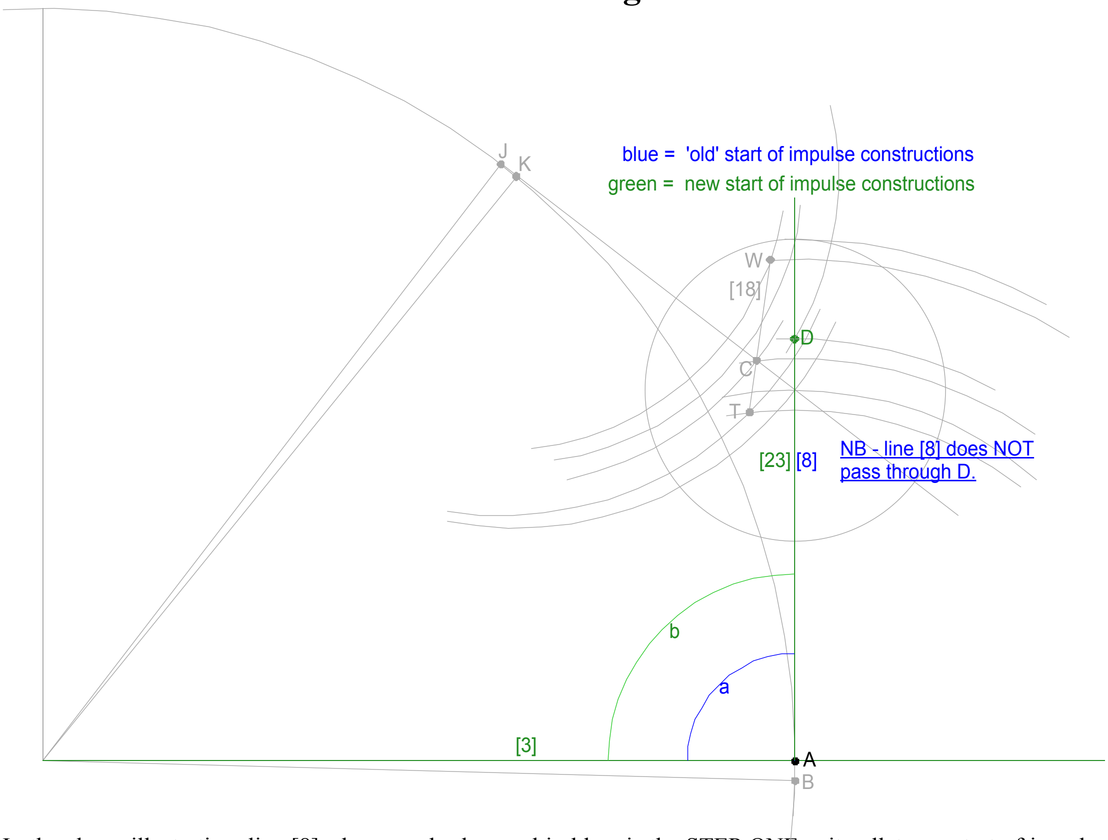
■ **[20] - Construct a circular arc (shown in red), centred at K, of the same radius as arc [19].** Potential final entry pallet arm pivot positions at the end of entry impulse lie along arc [20].

■ **[21] - Construct a circular arc (shown in red), centred at B, passing through point C.** The radius of this arc is the final active length of the exit pallet arm at the end of exit impulse.

■ **[22] - Construct a circular arc (shown in green), centred at A, of the same radius as arc [21].** Potential final exit pallet arm pivot positions at the start of exit impulse lie along arc [22].

The intersection of red end of entry impulse arc [20] and green start of exit impulse arc [22] is labelled 'D'. For the active entry and exit pallet arm lengths defined above, D is the final single pallets pivot pin location corresponding to the end of entry impulse and the *simultaneous* start of exit impulse. Unsurprisingly, point D, which is derived from point C, fails to coincide with line [8], the original, designer-chosen pallet arm line of action at the start of exit impulse. The misalignment is small, but must be eliminated. Fortunately, the process is extremely straightforward, as explained in STEP FIVE, next.

STEP FIVE - Figure 39



In the above illustration, line [8], shown as broken and in blue, is the STEP ONE exit pallet arm start of impulse construction, passing through intersection A.

Recall that angle 'a' is the designer-chosen STEP ONE angle, between the designer-chosen exit pallet arm line of action at the start of exit impulse, labelled as line [8] (broken blue line) and line [3]. The purpose of angle 'a', which will also be referred to as the **'STEP ONE exit angle'**, will become clear later, during adjustments to the escaping arc.

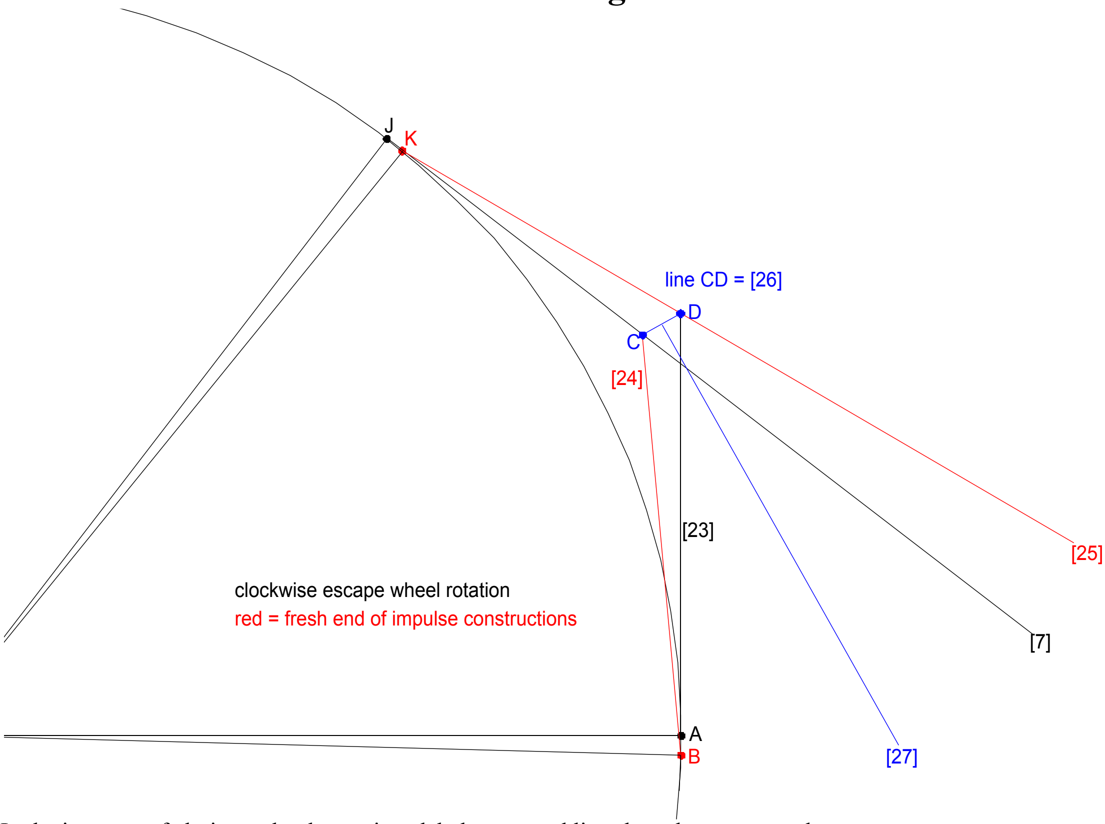
■ **[23] - Construct a straight line through A and D (shown above as a continuous green line).** Since line [8] is extremely close to line [23], it may be helpful to confirm that the green line [23] is anticlockwise removed from blue line [8]. Since point D is the final single pallets pivot pin position at the start of exit impulse for the CAD construction thus far, it follows that green line [23] must be the final exit pallet arm line of action at the start of exit impulse. Line [23] completely eliminates any deviation from truth, introduced by the incorrect assumption described in STEP FOUR, construction [18].

■ **Measure and record angle 'b' to as many decimal places as available software will permit** - Angle 'b' is the angle between the final exit pallet arm line of action at the start of exit impulse, labelled as line [23] (continuous green line) and line [3]. The purpose of angle 'b', which will also be referred to as the **'STEP FIVE exit angle'**, will become clear later, during adjustments to the escaping arc.

■ **Remove (blue) line [8].** Complete removal will eliminate potential confusion and incorrect CAD 'snap to' operations during subsequent steps of the design sequence. Line [8] has yet to be removed from the above illustration.

The illustrated difference between angles 'a' and 'b' is extremely small. Nevertheless, angle 'b' could be adjusted to almost match the original designer-chosen angle 'a' (if deemed necessary, for some reason) by simply altering the original angle 'a' in the appropriate sense by the difference between angles 'a' and 'b', returning to STEP ONE and repeating the entire CAD sequence up to this point. More precise adjustment of angle 'b' to match the original designer-chosen angle 'a' with any desired degree of precision could be achieved by repeating the described process as many times as necessary. No such adjustments have been made for this example. Be advised that any adjustments to achieve a designer-chosen escaping arc (explained later) would alter this angle, rendering the described exercise entirely pointless.

STEP SIX - Figure 40



In the interests of clarity, redundant points, labels, arcs and lines have been removed.

■ **[24] - Construct a line (shown in red) from C to B.** In preparation for later constructions, this adds the exit pallet arm line of action at the end of exit impulse.

■ **[25] - Construct a line (shown in red) from K to D and extend beyond D, approximately as shown.** In preparation for later constructions, this adds and extends the entry pallet arm line of action at the end of entry impulse.

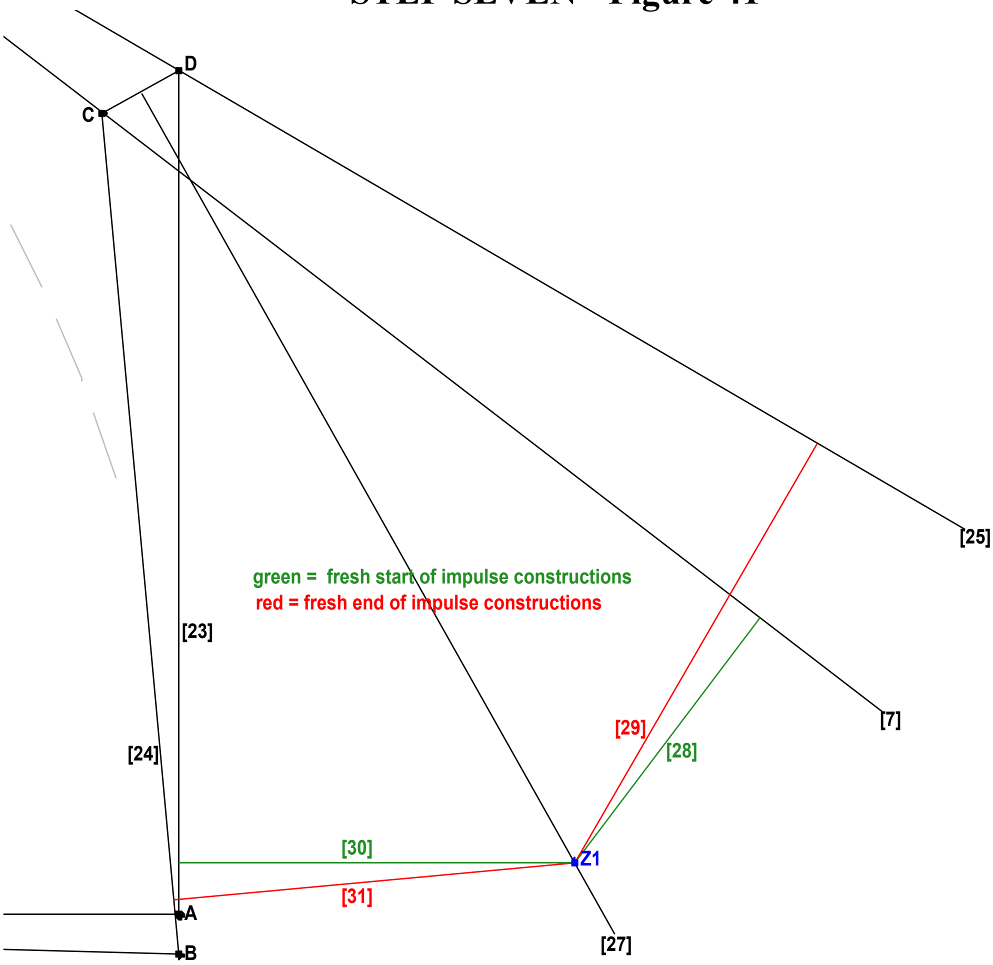
■ **[26] - Construct a line (shown in blue) from C to D.** Recall that C is the location of the single pallets pivot pin at the start of entry impulse and the end of exit impulse and D is the location of the single pallets pivot pin at the end of entry impulse and the start of exit impulse. Line [26] is essential to the next construction.

■ **[27] - Construct a straight line from the mid point of CD, perpendicular to CD.** This line, in principle extending to infinity, defines potential locations of the escapement frame arbor axis.

For the illustrated geometry, C and D are the only locations of the single pivot pin at which entry and exit pallet nib locking corners will be precisely captured and released by appropriate escape wheel teeth tips during each escapement cycle. C and D lie along the circumference of an imaginary circle, a small segment of which is traced by the single pallets pivot pin as the escapement frame pivots about the escapement frame arbor axis, located at the imaginary centre of the circle. Since CD is a chord of that circle, line [27] must be part of a radial of the circle, extending to the centre of the circle. Therefore, the escapement frame arbor axis must lie somewhere along line [27] if the escapement is to function (capture and release) correctly in continuous, repeating cycles. *Any* location of the escapement frame arbor along line [27] will guarantee correct escapement functioning.

For a given escapement frame arbor location along line [27], the escaping arc will be the angle subtended at the arbor axis by arc CD. The entry and exit pallet nib locking corners will be precisely captured and released by appropriate escape wheel teeth tips whenever the escapement frame (and any pendulum sharing the same arbor axis) is at either extremity of the escaping arc. Escapement frame arbor axis locations closer to C and D will obviously generate higher escaping arcs than locations further away from CD. However, it is extremely unlikely that a first CAD construction of line [27] will contain any location of the escapement frame arbor axis at which the designer-chosen escaping arc and the designer-chosen mean start/end torque arm ratio can be *simultaneously* achieved. Fortunately, the devised process of simultaneous incorporation is remarkably straightforward, as will be explained in due course.

STEP SEVEN - Figure 41



*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

■ **Z1 - Insert an arbitrary point, labelled 'Z1', coincident with line [27].** Fairly close to the illustrated location is advised, if only to avoid excessive future clutter. Z1 is a first trial location of the escapement frame arbor axis, the purpose of which will become clear later.

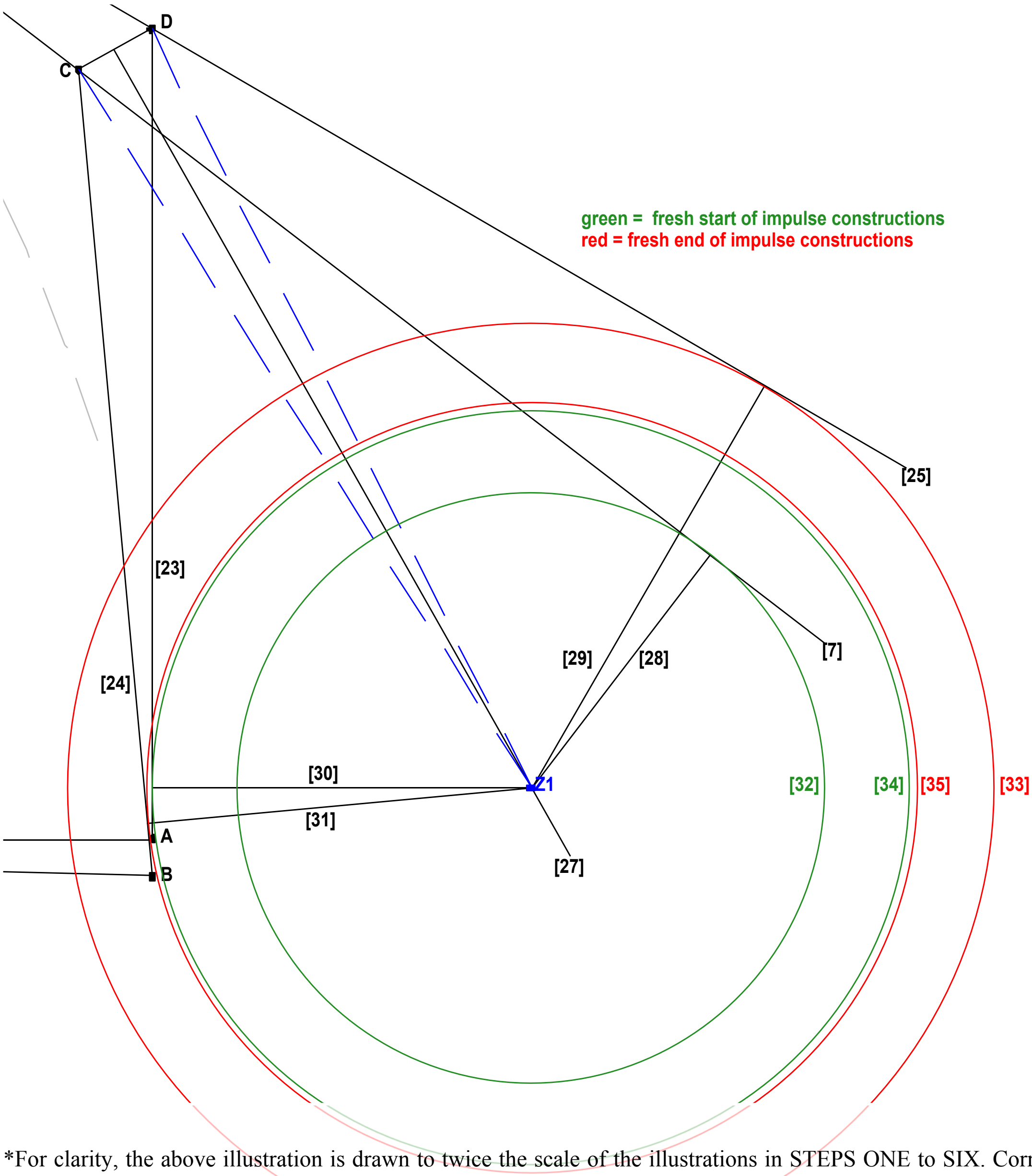
■ **[28] - Construct a line (shown in green) from Z1, perpendicular to and terminating at line [7].** Recall that line [7] is the designer-chosen entry pallet arm line of action at the start of entry impulse. For the chosen axis position, Z1, the length of this perpendicular (before the drawing scale was doubled*) is the torque arm of the escapement at the start of entry impulse.

■ **[29] - Construct a line (shown in red) from Z1, perpendicular to and terminating at line [25].** Recall that line [25] is the extended entry pallet arm line of action at the end of entry impulse. For the chosen axis position, Z1, the length of this perpendicular (before the drawing scale was doubled*) is the torque arm of the escapement at the end of entry impulse.

■ **[30] - Construct a line (shown in green) from Z1, perpendicular to and terminating at line [23].** Recall that line [23] is the exit pallet arm line of action at the start of exit impulse. For the chosen axis location, Z1, the length of this perpendicular (before the drawing scale was doubled*) is the torque arm of the escapement at the start of exit impulse.

■ **[31] - Construct a line (shown in red) from Z1, perpendicular to and terminating at line [24],** Recall that line [24] is the exit pallet arm line of action at the end of exit impulse. For the chosen axis position, Z1, the length of this perpendicular (before the drawing scale was doubled*) is the torque arm of the escapement at the end of exit impulse.

STEP EIGHT - Figure 42

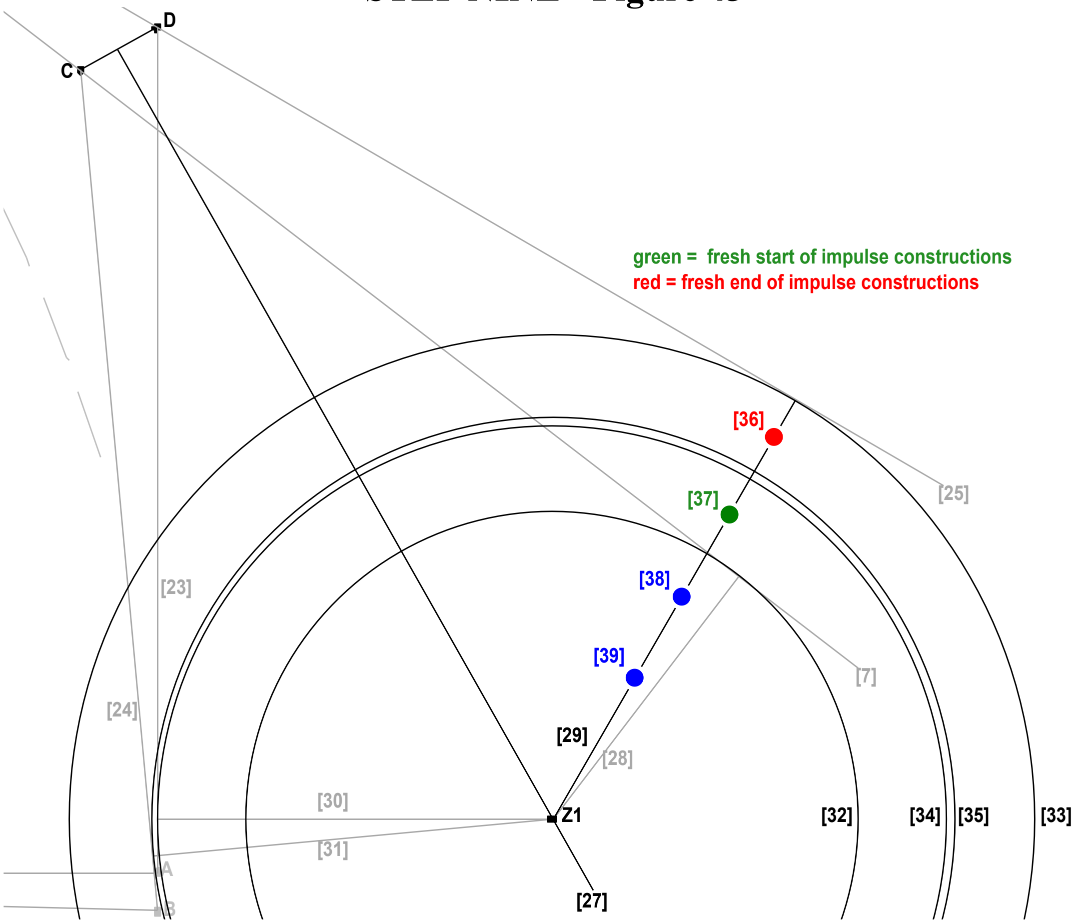


*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

■ **[32] - Construct a circle (shown in green), centred at Z1, of radius equal to the length of line [28].** For the chosen axis Z1 (before the drawing scale was doubled*), this is the 'torque arm circle' of the escapement at the start of entry impulse.
■ **[33] - Construct a circle (shown in red), centred at Z1, of radius equal to the length of line [29].** For the chosen axis Z1, (before the drawing scale was doubled*), this is the 'torque arm circle' of the escapement at the end of entry impulse.
■ **[34] - Construct a circle (shown in green), centred at Z1, of radius equal to the length of line [30].** For the chosen axis Z1, (before the drawing scale was doubled*), this is the 'torque arm circle' of the escapement at the start of exit impulse.
■ **[35] - Construct a circle (shown in red), centred at Z1, of radius equal to the length of line [31].** For the chosen axis Z1, (before the drawing scale was doubled*), this is the 'torque arm circle' of the escapement at the end of exit impulse.

Note carefully, for future reference, that since C and D are the extremities of normal single pivot pin motion and escapement frame arbor axis Z1 coincides with the (assumed) pendulum centre of motion, it follows that angle CZ1D (defined by broken blue lines) must be the achieved escaping arc for the STEP EIGHT construction.

STEP NINE - Figure 43



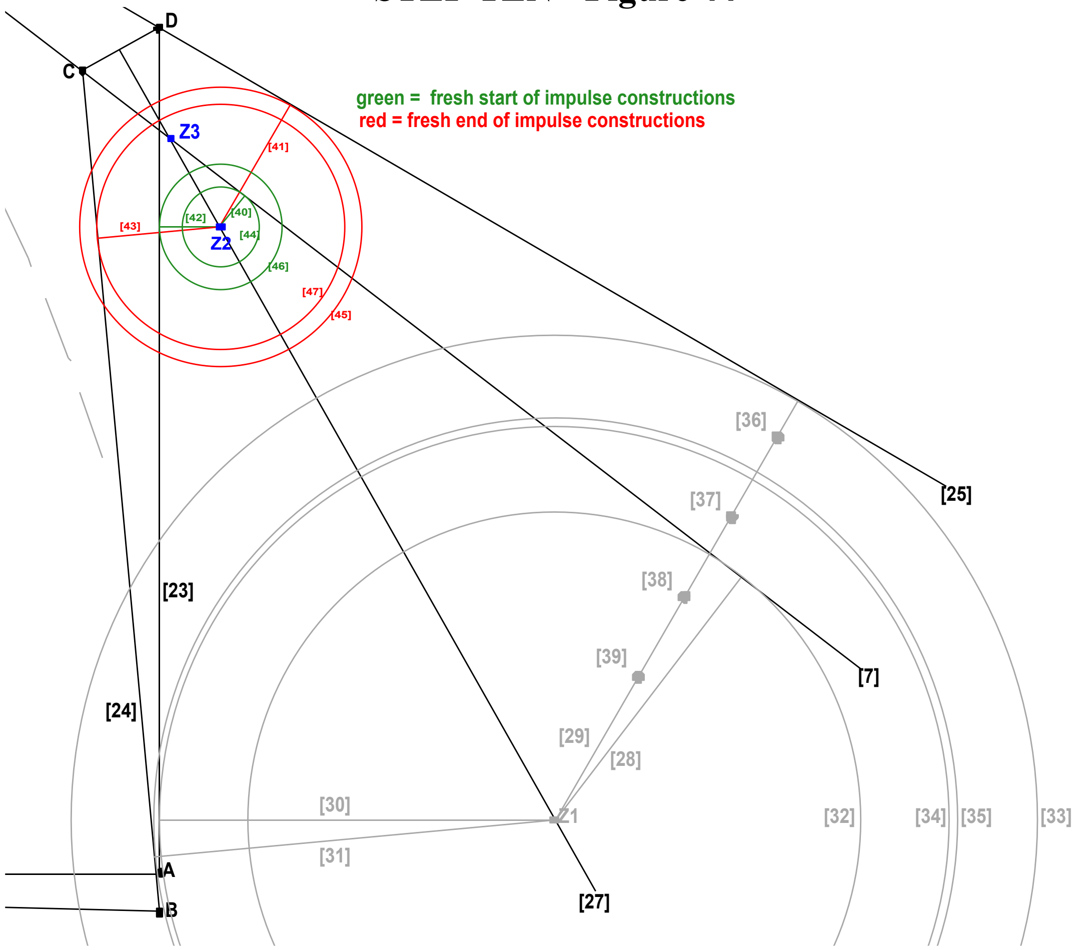
*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

NB - STEP NINE onwards will assume that the target, designer-chosen mean end/start ratio is 3 to 2, as Harrison stipulated. For any other choice of mean end/start ratio, a later section entitled 'GEOMETRICAL VARIATIONS' explains how the constructions should be adjusted.

■ **[36] - Insert a point (shown in red) on the the longest torque arm [29], equidistant from the arcs of the entry and exit end of impulse torque arm circles, [33] and [35].** The distance between Z1 and [36] (before the drawing scale was doubled*) is the mean end of impulse torque arm.
■ **[37] - Insert a point (shown in green) on the the longest torque arm [29], equidistant from the arcs of the entry and exit start of impulse torque arm circles [32] and [34].** The distance between Z1 and [37] (before the drawing scale was doubled*) is the mean start of impulse torque arm.
■ **[38] - Insert a point (shown in blue) on the the longest torque arm [29], which is the same distance from point [37] as point [36] is from point [37].** Distance [37] to [38] thereby equals distance [36] to [37].
■ **[39] - Insert a point (shown in blue) on the the longest torque arm [29], which is the same distance from point [38] as point [36] is from point [37].** Distance [38] to [39] thereby equals distance [36] to [37].

The first objective of the above constructions is to ensure that any two adjacent points from the four points [36], [37], [38] and [39] are equally spaced. The second objective is to ensure that the spacing of any two adjacent points is equal to the difference between the mean start of impulse torque arm and the mean end of impulse torque arm (before the drawing scale was doubled*). By achieving those two objectives, points [36], [37], [38] and [39] match Harrison's MS3972/3 layout of equally spaced points for a mean end/start ratio of precisely 3 to 2, as was illustrated in Figure 34.
However, in the above construction, point [39] is the necessary escapement frame arbor axis location for a mean end/start ratio of 3 to 2. Since point [39] fails to coincide with line [27], an escapement geometry based upon constructions [36], [37], [38] and [39] would certainly malfunction, because the pallet nib locking corners would fail to be accurately captured or released by the applicable escape wheel tooth tips (see the explanation in STEP SIX, for construction [27], should you seek a reminder). A straightforward solution will be revealed in due course.

STEP TEN - Figure 44



*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

This step duplicates STEPS SEVEN and EIGHT, with the significant exception that it is based upon a markedly different, second trial location of the escapement frame arbor axis location, labelled 'Z2'. Explanations will, therefore, be abbreviated or ommitted

■ **Z2 - Insert an arbitrary point, labelled 'Z2', coincident with line [27].** Z2 is the second trial location of the escapement frame arbor axis. Fairly close to the illustrated location is advised, if only to avoid excessive future clutter. In the interests of clarity and simplicity, avoid Z2 locations very close to the intersection labelled 'Z3' or anywhere between Z3 and line CD.

■ **[40] - Construct a (green) torque arm* from Z2, perpendicular to and terminating at line [7].**

■ **[41] - Construct a (red) torque arm* from Z2, perpendicular to and terminating at line [25].**

■ **[42] - Construct a (green) torque arm* from Z2, perpendicular to and terminating at line [23].**

■ **[43] - Construct a (red) torque arm* from Z2, perpendicular to and terminating at line 24].**

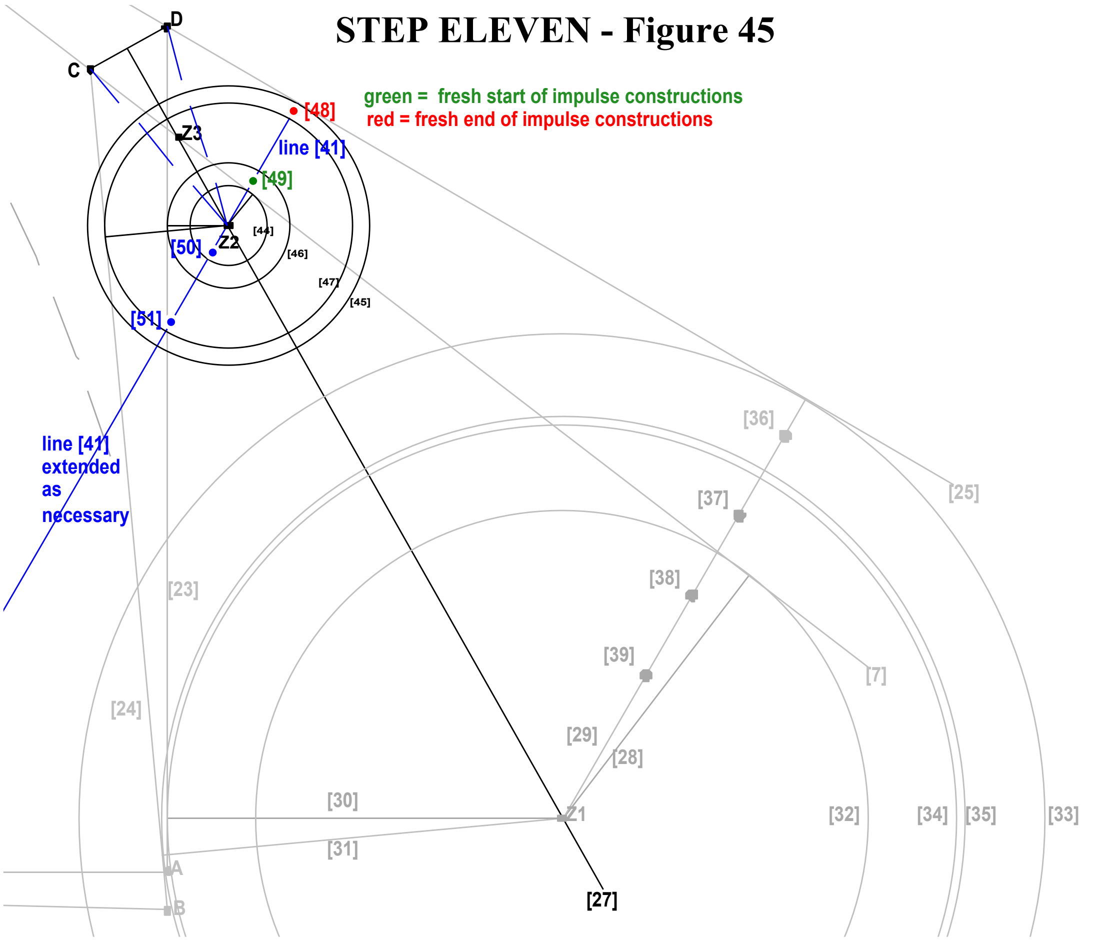
■ **[44] - Construct a (green) torque arm circle*, centred at Z2, of radius equal to [40].**

■ **[45] - Construct a (red) torque arm circle*, centred at Z2, of radius equal to [41].**

■ **[46] - Construct a (green) torque arm circle*, centred at Z2, of radius equal to [42].**

■ **[47] - Construct a (red) torque arm circle*, centred at Z2, of radius equal to [43].**

STEP ELEVEN - Figure 45



*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

This step duplicates STEP NINE, with the significant exception that it is based upon STEP TEN escapement frame arbor location Z2, instead of Z1. Explanations will, therefore, be abbreviated or ommitted.

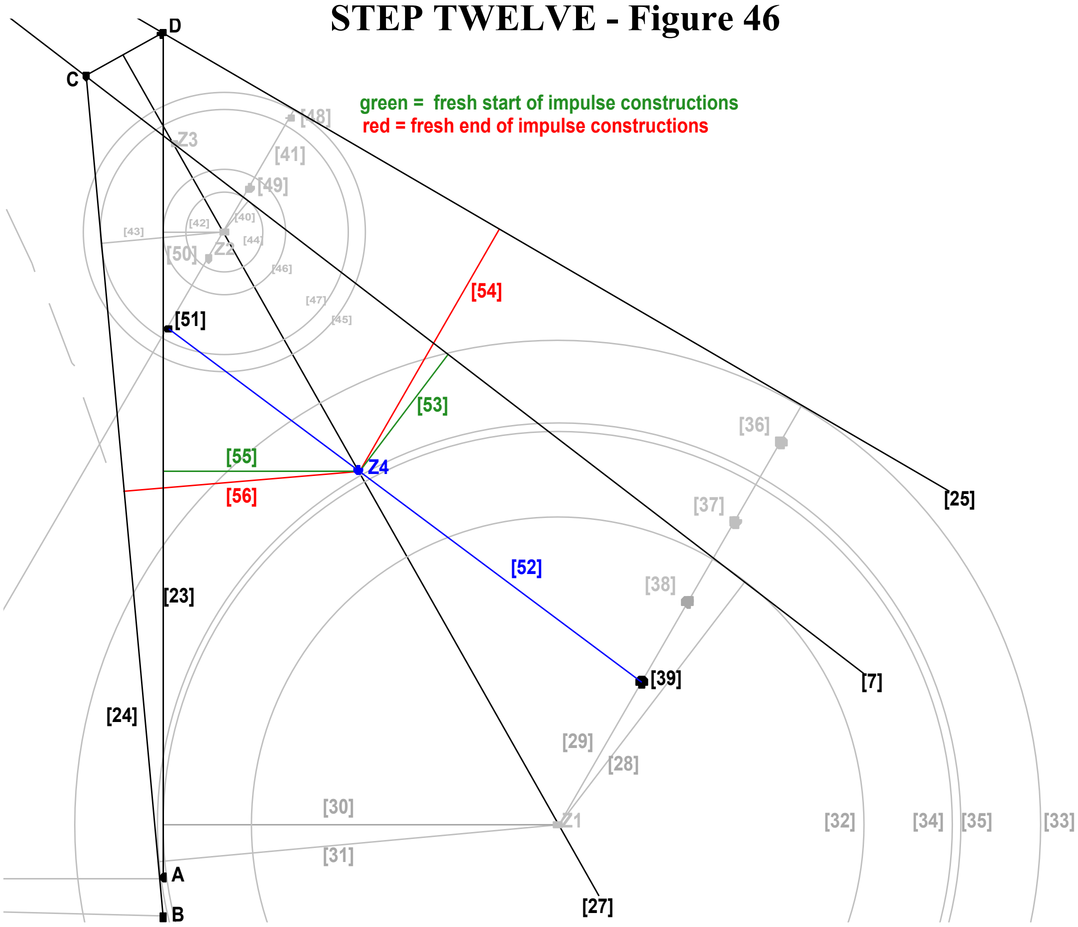
■ [48] - Insert a (red) point on the the longest torque arm [41], equidistant from the arcs of the entry and exit end of impulse torque arm circles, [45] and [47].
■ [49] - Insert a (green) point on the the longest torque arm [41], equidistant from the arcs of the entry and exit start of impulse torque arm circles, [44] and [46].
■ [50] - Insert a (blue) point on the longest torque arm [41], which is the same distance from point [49] as point [49] is from point [48]. If necessary, extend the longest (blue) torque arm [41], as shown.
■ [51] - Insert a (blue) point on the the longest torque arm [41], which is the same distance from point [50] as point [49] is from point [48]. If necessary, extend the longest (blue) torque arm [41], as shown.

The first objective of the above constructions is to ensure that any two adjacent points from [48], [49], [50] and [51] are equally spaced. A second objective is to ensure that the spacing of any two adjacent points is equal to the difference between the mean start of impulse torque arm and the mean end of impulse torque arm (before the drawing scale was doubled*). By achieving those two objectives, points [48], [49], [50] and [51] match Harrison's MS3972/3 layout of equally spaced points for a mean end/start ratio of precisely 3 to 2, as illustrated in Figure 34.

However, in the above construction, point [51] is the necessary escapement frame arbor axis location for a mean end/start ratio of 3 to 2. Since point [51] fails to coincide with line [27], an escapement geometry based upon constructions [48], [49], [50] and [51] would certainly malfunction, because the pallet nib locking corners would fail to be accurately captured or released by the applicable escape wheel tooth tips (see the explanation in STEP SIX, construction [27], should you require a reminder). A straightforward solution will be revealed in due course.

Note carefully, for future reference, that since C and D are the extremities of normal single pivot pin motion and escapement frame arbor axis Z2 coincides with the (assumed) pendulum centre of motion, it follows that angle CZ2D (defined by broken blue lines) must be the achieved escaping arc for the STEP ELEVEN construction.

STEP TWELVE - Figure 46



*For clarity, the above illustration is drawn to twice the scale of the illustrations in STEPS ONE to SIX. Correct torque arms are therefore half as long as those shown, notwithstanding the illustrative nature of all drawings.

■ **[52] - Construct a straight line (shown in blue) joining point [39] to point [51].** Line [52] and any extension thereof contains all possible locations of the escapement frame arbor (crutch arbor) axis for escapements generating a mean end/start ratio of 3 to 2 (easily verified, if desired, by considering alternative locations of Z1 and Z2).

■ **Z4 - Point Z4 (shown in blue) is at the intersection of lines [52] and [27]. If line [52] fails to intersect line [27], extend line [52] as necessary.** Point Z4 is the unique location of the escapement frame arbor (crutch arbor) axis guaranteeing correct mechanical functioning (by virtue of lying along line [27]), whilst *simultaneously* generating a mean end/start ratio of 3 to 2 (by virtue of lying along line [52]).

■ **[53] - Construct a (green) torque arm (subjected to scaling*) from Z4, perpendicular to and terminating at the designer-chosen entry pallet arm line of action at the start of entry impulse, line [7].**

■ **[54] - Construct a (red) torque arm (subjected to scaling*) from Z4, perpendicular to and terminating at the extended entry pallet arm line of action at the end of entry impulse, line [25].**

■ **[55] - Construct a (green) torque arm (subjected to scaling*) from Z4, perpendicular to and terminating at the exit pallet arm line of action at the start of exit impulse, line [23].**

■ **[56] - Construct a (red) torque arm (subjected to scaling*) from Z4, perpendicular to and terminating at the exit pallet arm line of action at the end of exit impulse, line [24].**

■ **T - Check that the achieved mean end/start (torque arm) ratio, T, corresponding to escapement frame arbor axis location Z4, matches the designer-chosen target ratio.** T = lengths{[54]+[56]}/{[53]+[55]}, regardless of any consistently applied drawing scaling*. An alternative, graphical check is to apply the constructions explained in STEP NINE or STEP ELEVEN for point Z4, checking that the four points thereby established are equally spaced and that the appropriate point coincides with Z4.

■ **M - Determine the mean torque arm.** Subject to scaling*, measure all four torque arms and divide their sum by four. An alternative, graphical method is to apply the constructions explained in STEP NINE or STEP ELEVEN for point Z4 and measure the distance between Z4 and a point mid-way between the mean end of impulse torque arm point and the mean start of impulse torque arm point. Subject to scaling*, that distance is the mean torque arm.

■ **CZ4D - Measure the achieved escaping arc (angle CZ4D) and compare it to the designer-chosen target.** Any discrepancy, if considered excessive, will require adjustment, using the technique explained next.

ADJUSTING THE ESCAPING ARC

An essential feature of any grasshopper escapement design technique claimed to be comprehensive is a capacity to simultaneously incorporate every one of Harrison’s CSM and MS3972/3 stipulations, or any designer-chosen variations. Steps ONE to TWELVE of the CAD sequence have completed those tasks, with the notable exceptions of mean torque arm and escaping arc, neither of which currently comply with this designer's choices of 10 mm and 9.7500 degrees (4 dp). Incorporation of a designer-chosen mean torque arm is a simple scaling operation, most economically performed (and explained herein) at the end of the entire design process. Achievement of the designer-chosen escaping arc is now due and is explained below.

Before proceeding further, it must be clearly understood that escapement frame arbor locations along line [27] and further from CD than Z4 in STEP TWELVE will always generate a mean end/start ratio lower than the achieved 3 to 2 and an escaping arc lower than CZ4D, whereas functional escapement frame arbor locations along line [27] and closer to CD than Z4 will always generate a mean end/start ratio higher than 3 to 2 and an escaping arc higher than CZ4D. It is, therefore, impossible to adjust the escaping arc by relocating the escapement frame arbor along the illustrated line [27] in STEP TWELVE without deviating from the already achieved 3 to 2 mean end/start ratio. To incorporate the designer-chosen escaping arc whilst *simultaneously* preserving the achieved mean end/start ratio, almost the entire geometry must undergo controlled alteration. Be assured that the devised technique is far more straightforward than might at first be imagined.

Adjustment of the escaping arc is based upon an identified correlation between escaping arc and the STEP FIVE exit angle (angle 'b' in STEP FIVE). This possibility was briefly mentioned during the earlier MS3972/3 analysis, when it was observed that exit lines of action at the start of impulse appeared to deviate very slightly from tangential in both single pivot geometries. It was considered especially relevant that Harrison highlighted long extensions to the exit lines of action in both cases, painstakingly drawing each as a series of closely spaced dots and it was thought unlikely that such emphasis and time consuming drawing effort would have been made and expended to no purpose. In strong support of those observations, detailed investigations have revealed that the achieved escaping arc is related to (and extremely responsive to) alterations in the STEP FIVE exit angle. Unfortunately, direct manipulation of the STEP FIVE exit angle can introduce considerable complication. For that reason, amongst others, a relationship between STEP FIVE exit angle and the designer-chosen STEP ONE exit angle (angle 'a' in STEP ONE) was imposed by circle [13] in STEP THREE. Escaping arc may thereby be adjusted by directly manipulating the STEP ONE exit angle, which is, of course, extremely straightforward.

In common with steps ONE to TWELVE of the CAD sequence, it will be convenient to illustrate the escaping arc adjustment process for a 120 tooth escape wheel spanning 17.5 mean tooth spaces. **Figures 47, 48 and 49** show three entirely separate geometries, each with a mean end/start ratio of 3 to 2, all created by following steps ONE to TWELVE of the CAD sequence for increasing STEP ONE exit angles. The achieved escaping arc increases from e1, through e2 to e3 as the input STEP ONE exit angle is increased from a1, through a2 to a3 respectively. In the interests of economical analysis, it is suggested that angle a2 be made equal the designer-chosen STEP ONE exit angle (for this example 90 degrees) and that angles a1 and a3 be five degrees below and above a2, respectively (for this example, therefore, 85 and 95 degrees respectively). Straightforward interpolation will identify the STEP ONE exit angle corresponding to the designer-chosen escaping arc. Graphical interpolation is an option, quite suited to the use of CAD. Simply plot escaping arcs e1, e2 and e3 versus exit angles a1, a2 and a3 respectively and fit a sensible curve to the plotted points. Input of the target escaping arc will identify the corresponding STEP ONE exit angle, albeit with an accuracy subject to the chosen resolutions of the arcs and angles and the truth of the curve fit. For practical purposes, a single interpolation will almost certainly be adequate. If considered necessary, more precise interpolation may be achieved by producing CAD geometries for STEP ONE exit angles very close to and to either side of the exit angle from the previous interpolation. Increasing precision may be achieved by repeating that process.

If the final STEP FIVE exit angle deviates from 90 degrees by more than MS3972/3 suggests is acceptable (a maximum of two degrees is proposed), it will be necessary to alter the mean tooth spaces spanned and repeat the entire design process from the beginning of STEP ONE. In such a situation, it will be useful to note that, for a given escape wheel tooth count and STEP ONE exit angle, escaping arc increases as the mean number of tooth spaces spanned increases and vice versa.

Figure 50 illustrates the arc-adjusted STEP TWELVE, 17.5 mean span geometry, delivering a designer-chosen escaping arc of 9.7500 degrees (4 dp), whilst simultaneously generating a mean end/start ratio of 3 to 2. The corresponding STEP FIVE exit angle is slightly greater than 88 degrees, which is considered acceptable. Note that the escape wheel PCD is still 200 mm, which will be dealt with shortly, when the mean torque arm is adjusted.

It may be of interest to learn that a repetition of the entire design process for the other MS3972/3 single pivot geometry, spanning 16.5 mean tooth spaces, results in a STEP FIVE exit angle of slightly less than ninety one degrees for a designer-chosen STEP ONE exit angle of exactly ninety degrees. An escaping arc of 9.7500 degrees and a mean end/start ratio of 3 to 2 match the 17.5 mean span geometry arc and ratio.

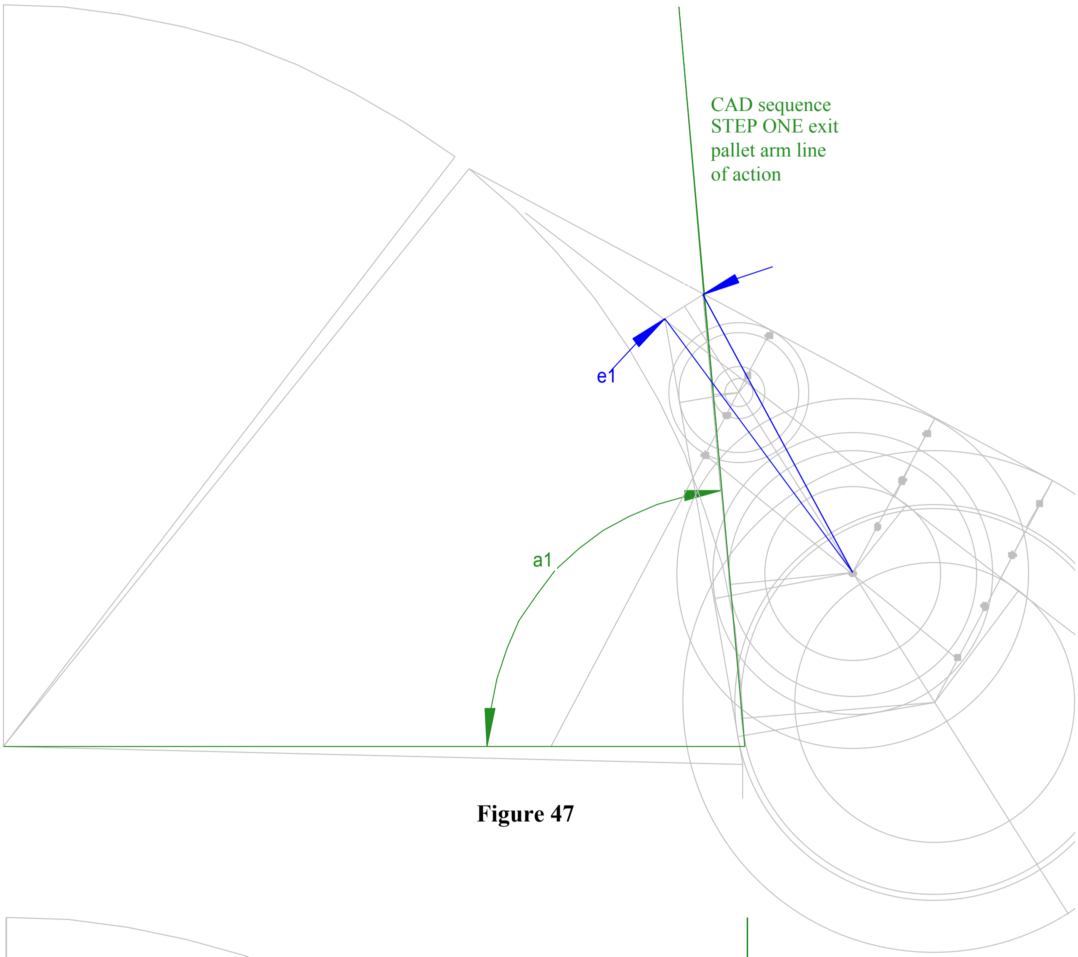


Figure 47

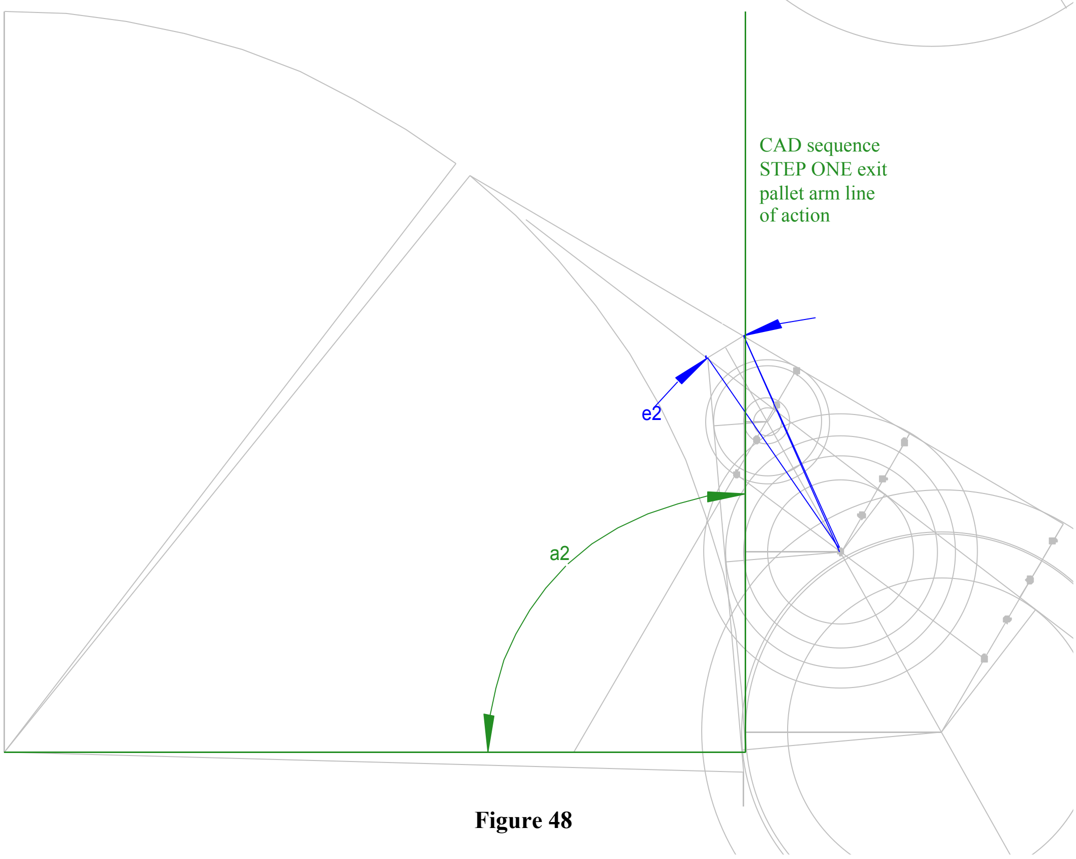
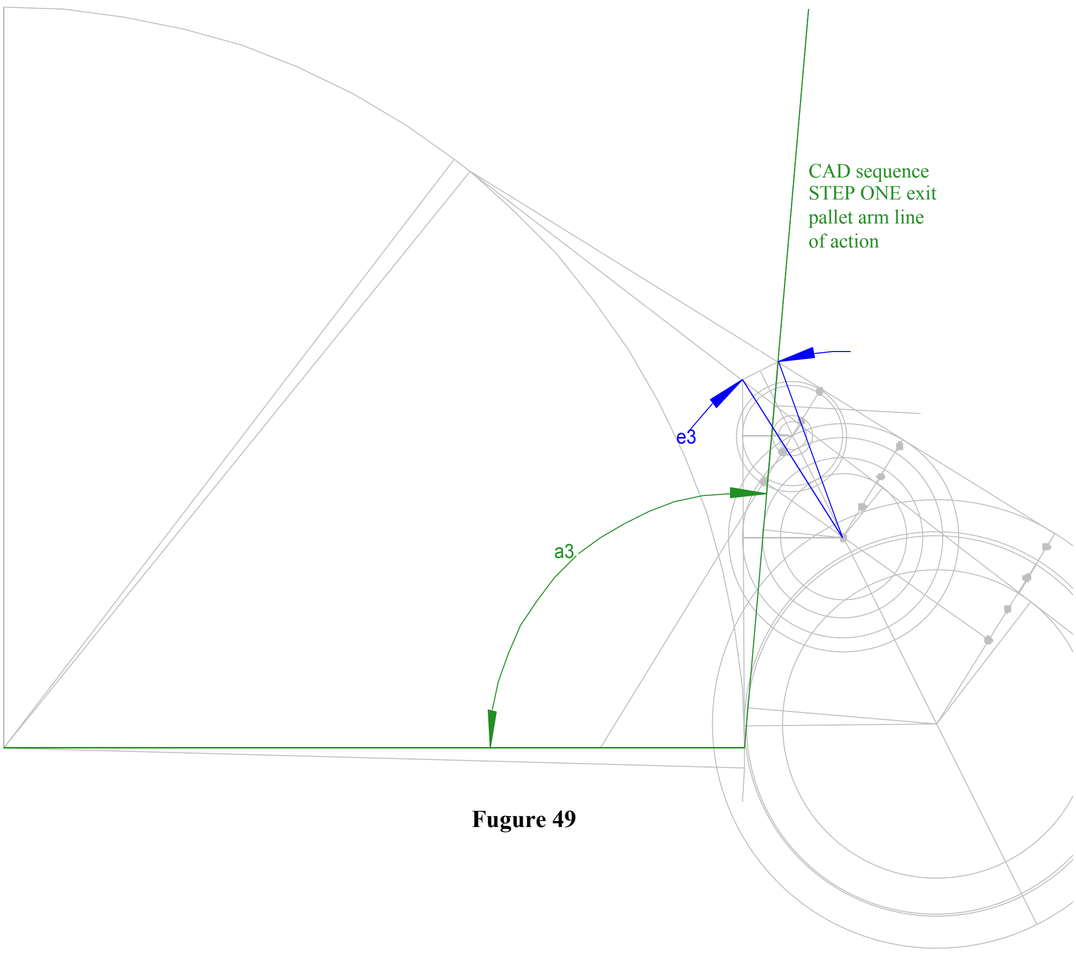


Figure 48



Fugure 49

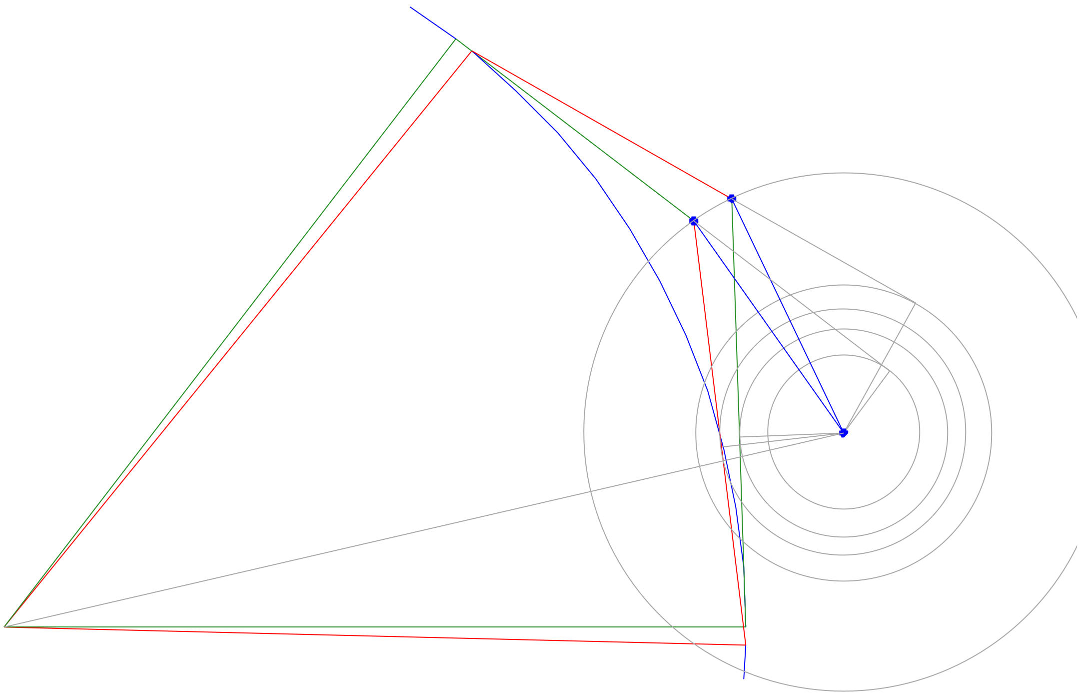


Figure 50 - 120 escape wheel teeth, 17.5 mean tooth spaces spanned geometry.
Mean end/start torque arm ratio 3 to 2, escaping arc 9.75 degrees, escape wheel PCD 200 mm.

ADJUSTING THE MEAN TORQUE ARM

The designer-chosen mean torque arm, for this example 10 mm, should now be incorporated. All linear dimensions of the *arc-adjusted* STEP TWELVE geometry should be scaled by the ratio 10 to M, where M is the measured mean torque arm from the arc-adjusted STEP TWELVE geometry, in this example for a 200 mm escape wheel PCD. This procedure may be completed in seconds, using the CAD scaling function. **Figure 51** shows the resultant geometry.

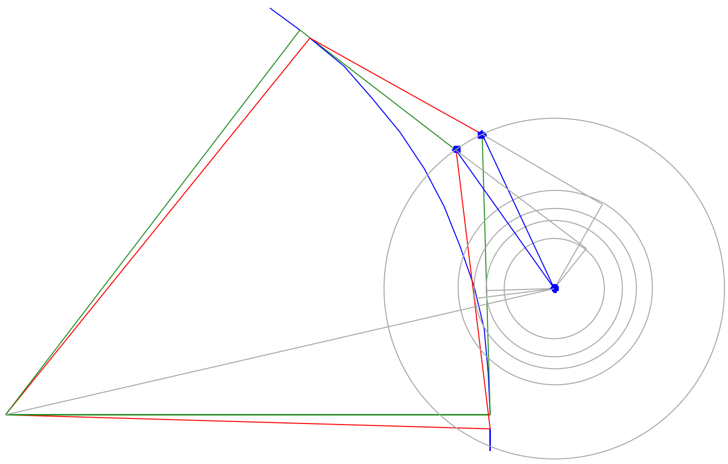


Figure 51 - 120 escape wheel teeth, 17.5 mean tooth spaces spanned geometry.
Mean end/start torque arm ratio 3 to 2, escaping arc 9.75 degrees, mean torque arm 10 mm.

DETERMINING PALLET NIB LIFTS

Although not a documented Harrison stipulation, the pallet nib locking corner lifts after release from the escape wheel (assumed to occur instantaneously) should be checked, by constructing circular arcs through J centred at Z, through K centred at D, through A centred at Z and through B centred at C, as shown in **Figure 52**. The appropriate intersections define the entry pallet nib locking corner lift, KV and the exit pallet nib locking corner lift, BW. The influence of instantaneous exit pallet nib locking corner lift will be discussed in detail shortly.

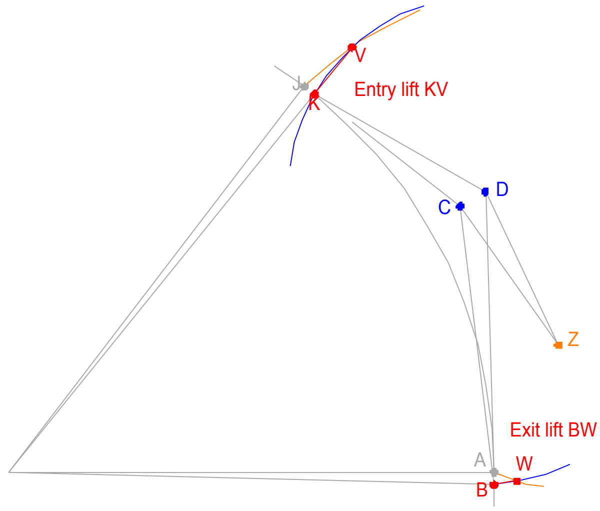


Figure 52 - Instantaneous pallet nib locking corner lifts after release.

CHECKING GEOMETRIES

Upon completion, or at any time beforehand if considered necessary, geometries should be checked.

With reference to **Figure 53** (drawn to no particular scale, with escape wheel axis 'O' out of illustrated view), the following checklist might be useful:

Check (1) Angle AOB should be half a tooth space subtended angle.
Check (2) Angle AOK should equal the minimum tooth spaces spanned subtended angle.
Check (3) Angle BOJ should equal the maximum tooth spaces spanned subtended angle.
Check (4) Angle JOK should be half a tooth space subtended angle.
Check (5) CJ should equal DK. Verifies equal entry pallet arm active lengths at the start and end of impulse.
Check (6) AD should equal BC. Verifies equal exit pallet arm active lengths at the start and end of impulse. This check, in combination with check (5), also verifies accurate locking corners capture and release.
Check (7) Angle CJO should equal the designer-chosen entry pallet arm line of action at the start of entry impulse.
Check (8) Angle DAO should be very close to the designer-chosen exit pallet arm line of action at the start of exit impulse. Within two degrees of 90 degrees is proposed.
Check (9) (GZ + MZ) / (FZ + LZ) should match the designer-chosen mean end/start ratio, T. For Harrison compliant geometries, this ratio must be 3 to 2, to the chosen degree of precision.
Check (10) 0.25 (FZ + GZ + LZ + MZ) should match the designer chosen mean torque arm, M.
Check (11) Angle CZD should match the designer-chosen escaping arc, to the chosen degree of precision.

If preferred, checks (9) and (10) may be achieved by drawing methods, rather than by measurement and calculation, as explained in STEP TWELVE.

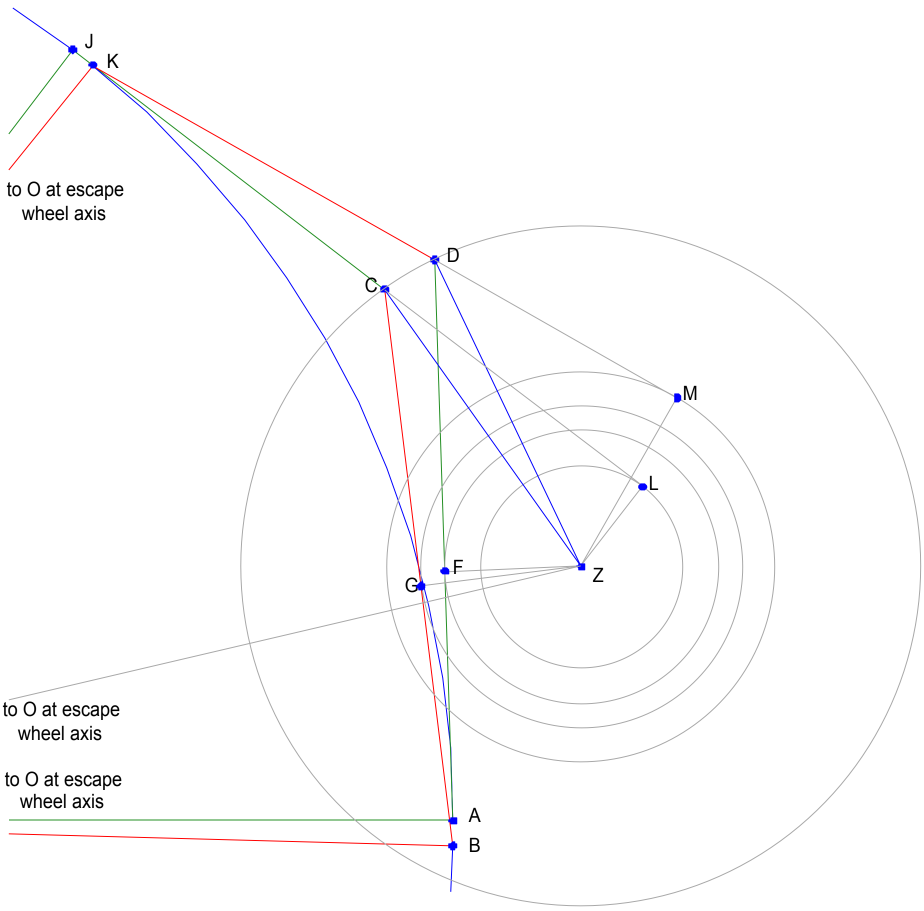


Figure 53

MS3972/3 REVISITED

Figure 54 illustrates the single pivot configurations of MS3972/3, spanning means of 16.5 and 17.5 tooth spaces, each derived from the CAD design sequence. The 16.5 mean span geometry is in red and the 17.5 mean span geometry is in blue. The geometries have been superimposed and aligned at their escape wheel centres and along their entry radials. In either case, the escape wheel contains 120 teeth and the escapement incorporates tangential entry and near-tangential exit lines of action, a mean end/start ratio of 3 to 2, a mean torque arm of 10 mm and an escaping arc of 9.7500 degrees (4 dp). It is clear that the two geometries are almost identical, apart from the differences in mean tooth spaces spanned and exit angles. A comparison of instantaneous exit pallet nib lifts (not shown, but discussed in detail later) also reveals insignificant differences, for all practical purposes. It is, therefore, possible that Harrison illustrated both geometries, because he had no valid reason to promote one in favour of the other.

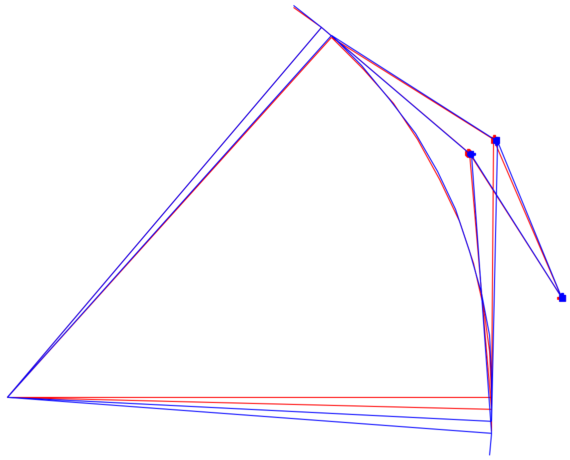


**Figure 54 - 120 tooth, 16.5 mean span geometry (in red) and 17.5 mean span geometry (in blue).
Mean end/start ratio 3/2, escaping arc 9.7500 degrees and mean torque arm 10 mm for both geometries.**

Investigations of mean spans other than 16.5 and 17.5 reveal that compliance with the majority of Harrison's stipulations is only possible if exit angles are allowed diverge from 90 degrees to an unacceptable extent, or if escaping arc is allowed to deviate markedly from the 9.75 degrees of MS3972/3. It would therefore appear that, for Harrison's identified constraints, mean spans of 16.5 and 17.5 tooth spaces are the only entirely compliant options.

Given the above conclusion, it is significant that the 16.5 and 17.5 mean span single pivot geometries illustrated in MS3972/3 are associated with different escape wheel PCD. In contradiction, Figure 54 demonstrates that when both geometries are scaled to a common mean torque arm, the escape wheel PCD are almost identical. It may be that, had it been published, MS3972/3 and its promised explanation would have served to demonstrate (amongst other things) how a specific mean torque arm, arbor separation, escape wheel PCD or pendulum length could be incorporated, by scaling an entire escapement geometry. It may also be that Harrison was illustrating the chosen escapement for his contemporary, unfinished, Final Regulator (commonly referred to as the RAS Regulator). In support of such a possibility, CSM includes the following statement: *'And now, if the Royal society please, I will show them the Draught of the Clock which I have in great Part made, and not only the Draught of the Pallats, as in particular, but also the Pallats themselves, in order that they may see at least some Reason for what I found, or might as in Consequence find from such a Contrivance of Pallats...'*. According to current, somewhat limited information* the 17.5 mean span geometry would appear to be Harrison's choice for the movement of his Final Regulator (the RAS Regulator).

In view of the above, it is interesting that Harrison's Final Regulator, on display at the Royal Observatory, Greenwich, England, currently incorporates a single pivot grasshopper escapement spanning an astonishing 22.5 mean tooth spaces of a 120 tooth escape wheel. It would also appear, from remote physical inspection, that the exit angle of the installed escapement deviates from the near-tangential constraint suggested by MS3972/3 to an considerable extent, whilst the extensive length of the markedly curved exit pallet arm absorbs a great deal, if not all, of the excess in mean span. By virtue of those deformities, exit pallet nib lift is appreciable and exit impulse is weakened (of which more later). Since the publication of CSM and the likely creation of MS3972/3 coincided with or followed the (incomplete) construction of Harrison's final regulator, it is unlikely that Harrison would have incorporated a mean span other than 16.5 or 17.5. Based upon current information, it can only be concluded that the 22.5 mean span escapement geometry is, at least in part, not by Harrison, but was a later victim of 'restoration' by someone completely ignorant of Harrison's single pivot grasshopper escapement design and performance principles.

*Although Final Regulator escapement measurements have reportedly been taken, a complete summary of dimensions has, at the time of writing, yet to be published. Upon publication, confirmation of much of the above (and more besides) will be extremely straightforward.

OBSERVATIONS

Figures 55 to 60 illustrate six single pivot geometries for escape wheels of 30, 60, 90, 120, 150 and 180 teeth, all scaled to a mean torque arm of 10 mm. Intermediate tooth counts are undesirable, in view of the likely requirement to indicate seconds from the escape wheel arbor (since there are sixty seconds in a minute, multiples of thirty teeth, beginning at thirty, are the only sensible choices for tooth count). The mean tooth spaces spanned was dictated by a nominally constant target escaping arc, a mean end/start ratio of 3 to 2 and tangential lines of action at the starts of impulse. Instantaneous entry and exit pallet arms lines of action after release and corresponding pallet nib lifts are essential inclusions.

TORQUE ARM CIRCLES

In all sampled cases, the presence of four separate torque arm circles is an indication of asymmetric entry vs exit torque arms. Asymmetric impulse is, in realistic terms, an unavoidable feature of the single pivot grasshopper escapement geometry, which Harrison dismissed as irrelevant, presumably conditional upon the correct and complete incorporation of his grasshopper escapement stipulations.

As escape wheel tooth count increases, the torque arm circles for equivalent phases of the operating cycle (i.e. for either both starts of impulse, in green, or for both ends of impulse, in red) become increasingly similar, revealing a trend towards greater entry vs. exit symmetry. This suggests that, for tangential lines of action at the starts of impulse, perfectly symmetrical impulse would only be delivered by a single pivot grasshopper escapement incorporating an infinite number of escape wheel teeth.

Careless inspection might suggest that the difference between the radii of the two outer (end of impulse) torque arm circles matches the difference between the radii of the two inner (start of impulse) torque arm circles. Unsurprisingly, equality is not confirmed by measurement. The appearance merely stems from the incorporation of matching lines of action relative to the applicable escape wheel radials at the starts of impulse, the intersections of all four lines of action with one of two extremes of motion of the single pallets pivot pin and the symmetrical disposition of the escapement frame arbor relative to those two extremes of motion. The inequalities become slightly more apparent as tooth count decreases and progressively more obvious as the orientations of entry and exit start of impulse lines of action relative to the applicable escape wheel radials increasingly differ.

ESCAPE WHEEL TOOTH COUNT

An obvious effect of raising the escape wheel tooth count is an increase in overall escapement size. In all cases, a greater escape wheel PCD is a significant contributor. Although the higher tooth counts would appear to generate inconveniently large escapements, cantilever installation behind the rear main plate of the movement alleviates many of the difficulties. Furthermore, if Harrison's radial flank wheel teeth and roller pinion gearing is incorporated, the inherently large pitch circle diameters within the train render escapement size less of a problem. Harrison's Final Regulator is an especially fine example of what can be achieved. Most significantly, the increase in escape wheel PCD with increasing escape wheel tooth count serves to maintain an almost constant spacing between adjacent escape wheel teeth tips.

Performance considerations exert a considerable influence upon the choice of tooth count. In CSM Harrison emphasises the importance of avoiding low torque to the escape wheel arbor. His sound reasoning is that any variations in delivered torque will have a greater proportional effect upon smaller escape wheel driving torques than they would upon larger torques. For that reason (albeit in isolation), for a given mean torque arm, as large a diameter escape wheel as possible is to be preferred, which translates into as high an escape wheel tooth count as possible. Escape wheels with less than Harrison's CSM stipulation of one complete revolution every four minutes (i.e. 120 teeth in combination with a seconds-beating pendulum) should therefore be avoided, if possible.

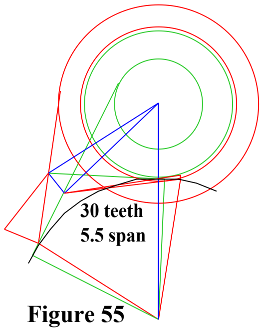


Figure 55

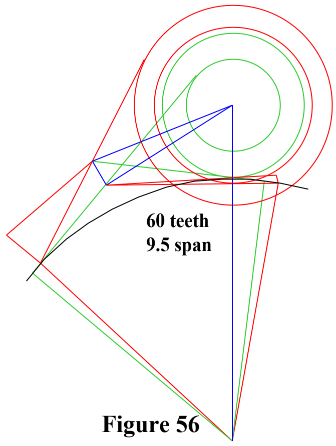


Figure 56

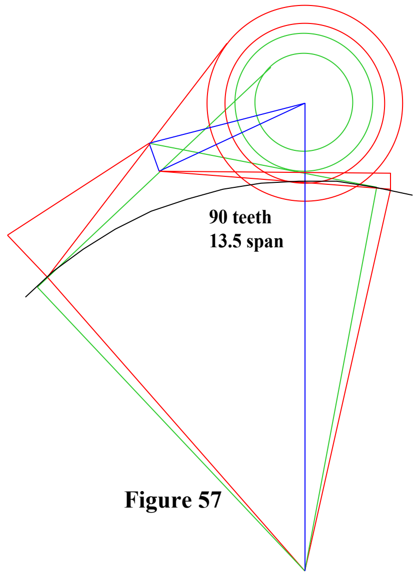


Figure 57

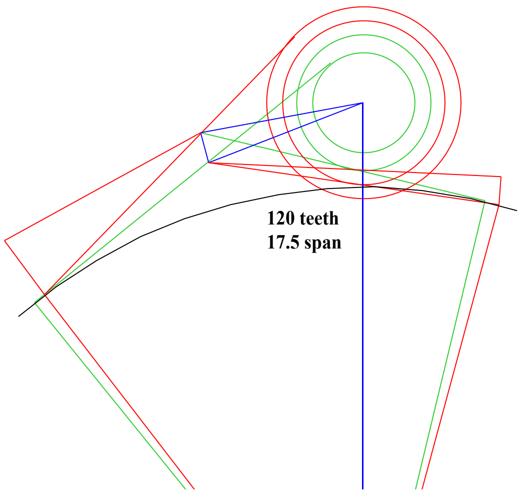


Figure 58

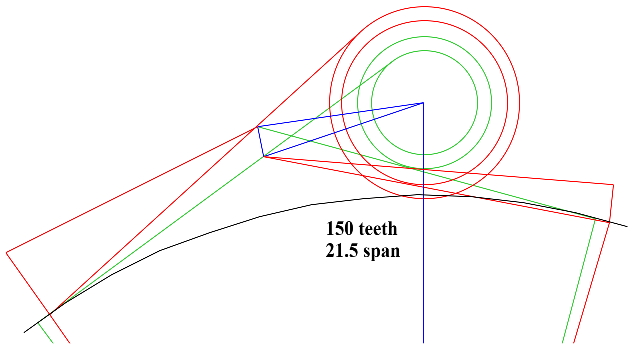


Figure 59

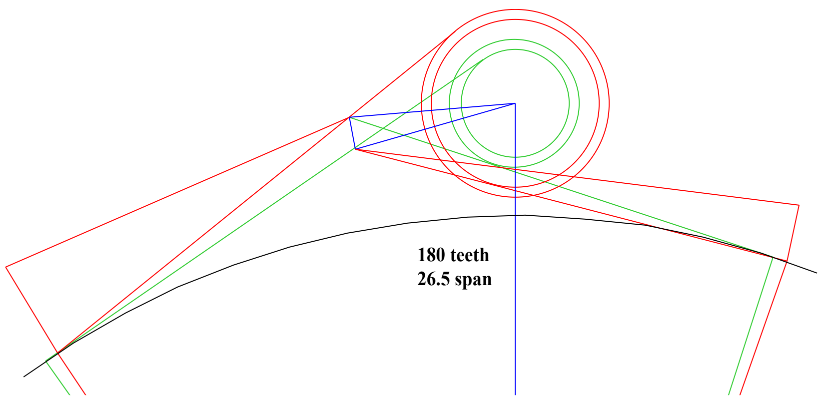


Figure 60

EXIT PALLET NIB LOCKING CORNER LIFT

The exit pallet nib lift and the length of the exit pallet nib can be extremely influential design considerations. For maximum trip protection, the designed exit pallet nib length should match the exit pallet nib lift, thereby minimising, in theory, any opportunities for the nib to completely clear the escape wheel (apart from the brief, essential period after release). It is apparent from the six sample geometries that instantaneous (i.e. zero pallet arm transit time) exit pallet nib lift becomes markedly greater with increasing escape wheel tooth count, from which it follows that if maximum escapement trip and escape wheel runaway protection is to be maintained, the exit pallet nib length must also increase as escape wheel tooth count increases. However, the profile between the escape wheel teeth must accommodate the exit nib during capture and overswing at the very least (ignoring any safety margins for unintended excesses of pendulum amplitude). As a consequence, a significant outcome of increasing tooth count is a deeper profile between escape wheel teeth, to accommodate increasing exit nib length. Unfortunately, as explained under ESCAPE WHEEL TOOTH COUNT, escape wheel tooth spacing is almost constant, regardless of tooth count. Therefore, a deeper profile between adjacent teeth unavoidably leads to a more slender escape wheel tooth of lower strength, a more fragile tooth tip and a reduced resistant to impact. Such considerations are of particular relevance during trip and runaway events, when high speed collisions between teeth tips and pallet nibs are a distinct and potentially damaging possibility. In such a situation, slender nibs and escape wheel teeth are an unwelcome liability.

A simple, direct and somewhat defeatist solution would be to accept reduced trip protection by incorporating a shorter exit nib, thereby permitting shorter, stronger escape wheel teeth. The more satisfactory course of action would be to preserve maximum trip protection and reduce the escape wheel tooth count until exit pallet nib lift allowed the incorporation of nibs and escape wheel teeth of adequate strength. With that in mind, tooth counts of 150 and above are increasingly less satisfactory than 120.

At the opposite extreme, an escapement geometry with a low tooth count and a correspondingly low exit pallet nib lift will require a short exit pallet nib length, a shallow gap between adjacent escape wheel teeth and, therefore, a strong escape wheel tooth profile. Unfortunately, reduced exit nib lift and nib length might, if taken too far, introduce troublesome sensitivity to manufacturing quality, adjustment and operating environment. The 30 and 60 teeth escapements of Figure 55 and 56 are examples of what should be avoided, most especially when account is also taken of the detrimental effects of low tooth count in other respects.

VARYING FORCES

The effect of varying forces is to generate a mean end/start ratio of delivered *torques* which deviates, numerically, from the designer-chosen mean end/start ratio of torque *arms*, so painstakingly incorporated within the geometry in accordance with Harrison's identified instructions. Pallet arm lines of action tangential to the escape wheel will transmit the greatest force, whilst non-tangential lines of action will reduce that force in proportion to the sine of the angle between the line of action and the escape wheel radial through the point of pallet nib locking corner capture. For tangential entry and exit lines of action at the start of impulse, the deviation of the end of impulse lines of action from tangential is greatest for the 30 tooth escape wheel, reducing as tooth count is increased. Although, as described much earlier, CSM reveals that Harrison was unconcerned by the effect of varying forces, it may be that his accompanying CSM stipulation of a four minute escape wheel (120 escape wheel teeth) was an essential condition. Would Harrison have accepted the more pronounced effect of varying forces for lesser tooth counts? A cautious designer would assume not, by specifying no less than 120 escape wheel teeth.

VARIATIONS

The inventive designer will inevitably seek to eliminate uncomfortable design compromises at source, as Harrison so often did with astonishing success. However, although some of the variations offered below are certainly feasible, it must be emphasised that any significant deviations from Harrison's CSM and MS3972/3 instructions are best avoided, pending a complete understanding of his land-based precision regulator science. Nevertheless, an exploration of a few variations is undeniably irresistible, usefully insightful and could be of some future value. If nothing else, the remarkable versatility of the CAD design technique will be demonstrated.

In preparation, it will be useful to recall that, in accordance with MS3972/3, the CAD design sequence was based entirely upon an entry pallet arm line of action at the start of entry impulse at all times tangential to the escape wheel PCD. The desired escaping arc was achieved by altering the STEP ONE exit angle at the start of exit impulse by no more than a proposed two degrees.

Variation 1 - The roles of the entry and exit lines of action may be reversed. Thus, the *exit* pallet arm line of action at the start of exit impulse would at all times be tangential and, in compensation, the STEP ONE *entry* pallet arm line of action at the start of entry impulse would automatically diverge from tangential during the escaping arc adjustment process. Harrison's remaining stipulations could otherwise still be incorporated. The comments within Variation 3, below, should be noted.

Variation 2 - Either pallet arm line of action at its start of impulse could be displaced from tangential by a fixed, designer-chosen amount and, in compensation, the STEP ONE line of action of the other pallet arm at its start of impulse would automatically diverge from tangential during the escaping arc adjustment process. The remainder of Harrison's stipulations could otherwise still be incorporated. This variation manipulates the entry side of the escapement relative to the exit side and could, for example, absorb excessive exit angle deviations without altering the mean tooth spaces spanned, by sharing the excess between each side of the escapement. The comments within Variation 3, below, should be noted.

Variation 3 - Variations 1 and 2 are perhaps moderately acceptable, provided that applicable STEP ONE angles are only allowed to deviate from 90 degrees by no more than a proposed two degrees. However, the incorporation of markedly non-tangential lines of action is a more blatant and potentially hazardous violation of Harrison's MS3972/3 instructions.

It is immediately obvious that there is little sense in providing torque to the escape wheel if a proportion of it is then squandered by incorporating markedly non-tangential lines of action. Tangential action is clearly the most efficient.

An acute line of action, undercutting the path of the escape wheel teeth tips, will demand a curved pallet arm, in order to clear the escape wheel teeth tips, which increases manufacturing difficulties and cost.

A balancing of entry vs. exit start of impulse pallet arms lines of action will restrict the effect of varying forces,. Notwithstanding the precise incorporation of a 3 to 2 mean end/start ratio of *torque arms* within the geometry, the greater the deviation of a pallet arm start of impulse line of action from tangential, the greater will be the reduction in transmitted forces on that side of the escapement. In terms of Harrison's intentions, that reduction, completely unaccounted for during the design of the geometry, would distort the physical delivery of impulse to the pendulum, unless both sides of the escapement effected the same reduction.

Despite all of the above, the flexible CAD design technique will accommodate any designer-chosen demands with ease.

Of passing interest (and ignoring the possibility that they may not be original fitments) Harrison's very early grasshopper escapements, fitted to movements constructed almost entirely of hardwood, are currently of the single pivot configuration. Although *reliable* information is surprisingly sparse, the escapements apparently incorporate oblique lines of action at the starts of impulse. It is however quite possible, if not likely, that, at such an early stage in Harrison's development of the grasshopper escapement, detailed performance and design stipulations had yet to be identified. Accurate nib locking corner capture and release events might well have been Harrison's only objectives at that time. Oblique lines of action also permit straight pallet arms to be fitted, easing construction.

Variation 4 - For a given designer-chosen escaping arc, the instantaneous exit pallet nib lift may be manipulated by increasing or decreasing the mean number of tooth spaces spanned. In such cases, notwithstanding other variations, the exit angle must also be adjusted, in order to restore the chosen escaping arc. An exit angle close to tangential might then be impossible for the altered span.

Variation 5 - The mean end/start ratio may be manipulated with ease, although this is another clear case of blatant disregard for a Harrison instruction, which clearly specifies a ratio of 3 to 2.

Figure 61 is an illustration, to no intentional scale or proportions, of the four torque arm circles of a single pivot grasshopper escapement. The two inner circles, in green, represent the start of impulse torque arms and the two outer circles, in red, represent the end of impulse torque arms. Z is the crutch arbor axis, XZ is the mean of the radii of the inner circles (and, therefore, the mean start of impulse torque arm) and WZ is the mean of the radii of the outer circles (and, therefore, the mean end of impulse torque arm).

A universal expression for the mean end/start ratio, T, is derived as follows:

T = WZ / XZ from the earlier MS3972/3 analysis

therefore T = (WX + XZ) / XZ

hence **XZ = WX / (T-1)**

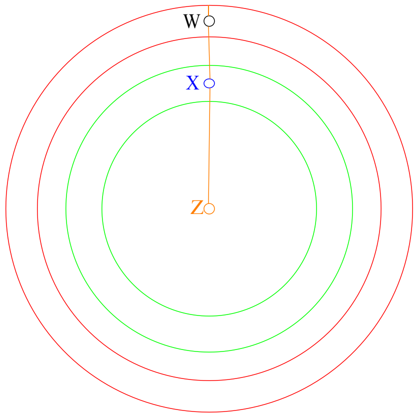


Figure 61

All CAD sequence constructions are unaltered, with the following exceptions:

STEP NINE - Omit point [38]. Point [39] is separated from point [37] by distance [36] to [37] divided by (T-1).

STEP ELEVEN - Omit point [50]. Point [51] is separated from point [49] by distance [48] to [49] divided by (T-1).

STEP TWELVE - 'T' - Calculated verification is unaltered. Graphical verification must be based upon the above.

Variation 6 - There may be a requirement to reproduce nothing more than the appearance and geometry of an existing escapement. Variation 3, for example, mentioned the escapement of Harrison's wooden movement regulators, which is a good example. In such cases, precise targeting and adjustment of mean end/start ratio and escaping arc must be abandoned. The achievement and verification of correct functioning are the only objectives. The former may be achieved by completing STEP ONE to STEP SIX inclusive of the CAD design sequence and omitting STEP SEVEN onwards, whilst the latter may be achieved by completing Checks (1) to (8) inclusive of the section entitled CHECKING GEOMETRIES.

Variation 7 onwards? - Six variations have been offered and it is likely that many more are possible. The only limitation is the designer's imagination, given the astonishingly flexible and capable CAD design technique. Note that mechanical considerations are beyond the scope of this publication. Modern materials, manufacturing processes and mechanical design variations etc. could significantly influence the design optimisation process and any conclusions derived from pure geometries alone.

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