PERFECTING THE HARRISON SINGLE PIVOT GRASSHOPPER ESCAPEMENT

Perfecting the Harrison Single Pivot Grasshopper Escapement

First Edition

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References to excluded parts of the parent publication have been removed, leaving several unavoidable gaps, in order to avoid disruption to the layout.

Figures retain their original numbers (6.01 to 6.51).

The term "replica" must not be taken to mean "exact copy".

This publication is not intended to be a thorough or complete constructional guide. Information has been supplied, in good faith, in order to assist construction by those with sufficient skills and experience.

All figures and diagrams are illustrative only.

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INTRODUCING THE SINGLE PIVOT GRASSHOPPER ESCAPEMENT

To those who are unfamiliar with the single pivot grasshopper escapement, a first sighting is both a puzzling and a captivating experience, in that order to some, but in an alternating kaleidoscope of such emotions to others. It is immediately obvious, to virtually any observer, that the escape wheel most certainly advances in regular steps, exactly as one might expect of 'clockwork'. Sure enough, the familiar pendulum sways from side to side with undiminished energy, as all pendulums do. But how on Earth is that gyrating, knotted puzzle of interwoven components, apparently acting as go-between, orchestrating such regular motions, when it would appear to lack any self-control? Disaster seems imminent. Yet disaster never strikes. The regular motions of the escape wheel and pendulum continue with undiminished vigour and unerring precision.

Observers possessed of sufficient patience are rewarded, sooner or later, with an understanding of how the trick is performed. In the fashion of an optical illusion, the gyrating puzzle, somehow, transforms itself into an ingenious solution, skilfully repeating an elegant, well rehearsed dance. Smiles appear on faces. Heads nod in appreciation and shake in amazement. For some, realisation strikes in seconds, whilst others struggle for longer. A few would never completely understand, were it not for some well chosen words of guidance.

If only one could incorporate a working grasshopper escapement into an publication such as this. How much easier the tasks of explanation and understanding would be. Indeed, how truly wonderful such a publication would be. As it is, a careful explanation of the cycle of operation of our skilful little acrobat will occupy considerably more than a few painstakingly compiled pages of illustrated text, if we are to ensure complete understanding and avoid any risk of confusion. Furthermore, a promise will be made, at the start of this journey, to strive for the enlightenment of an audience of widely varying aptitudes, from all walks of life. Fulfilment of that promise will demand particular care in explanation, not to mention patient understanding from those who are naturally quick to grasp such concepts.

MECHANICAL ARRANGEMENT

In contrast to, for example, the anchor and dead beat escapement, the mechanical arrangement of the single pivot grasshopper is somewhat more involved, although by no means difficult to understand. Nevertheless, for those who are new to mechanics and who find the brief description presented in this preliminary section to be somewhat difficult, please be assured that it is merely intended as an introduction and that a subsequent section, describing the complete operating cycle, will carefully explain and generously illustrate every introduced topic.

The proportions of the escapement used to illustrate the description will, to some extent, be a modified and refined version of those presented in an unfortunately obscure, untitled Harrison layout drawing, discovered amongst his possessions after his death. That layout was, quite possibly, an attempt to perfect the escapement geometry for his intended masterpiece of land-based precision timekeeping, his 'Final Regulator'. However, as will be appreciated much later in this tale, Harrison's manuscript, CSM, must be our more influential, definitive guide to what constitutes a perfect geometry.

Figure 6.01 (below) - Shows the optimised grasshopper escapement of Harrison's regulator, complete escape wheel and pendulum in simplified, somewhat illustrative form (for clarity), viewed from the front. The pendulum is a greatly abbreviated, symbolic representation of the physical reality, which would certainly be more than a metre in overall length. In comparison with the anchor and dead-beat, the grasshopper escapement shown incorporates a relatively large diameter escape wheel (in this case, for a seconds beating pendulum, the pitch circle diameter of the tooth tips is almost 145mm), with a correspondingly high number of teeth (one hundred and twenty in this case).

The escape wheel is typical of the type favoured by Harrison in his later work; his sea clocks, H2 and H3 and his Final Regulator all follow the same general form. The teeth are extremely slender and are, as a consequence, rather delicate.

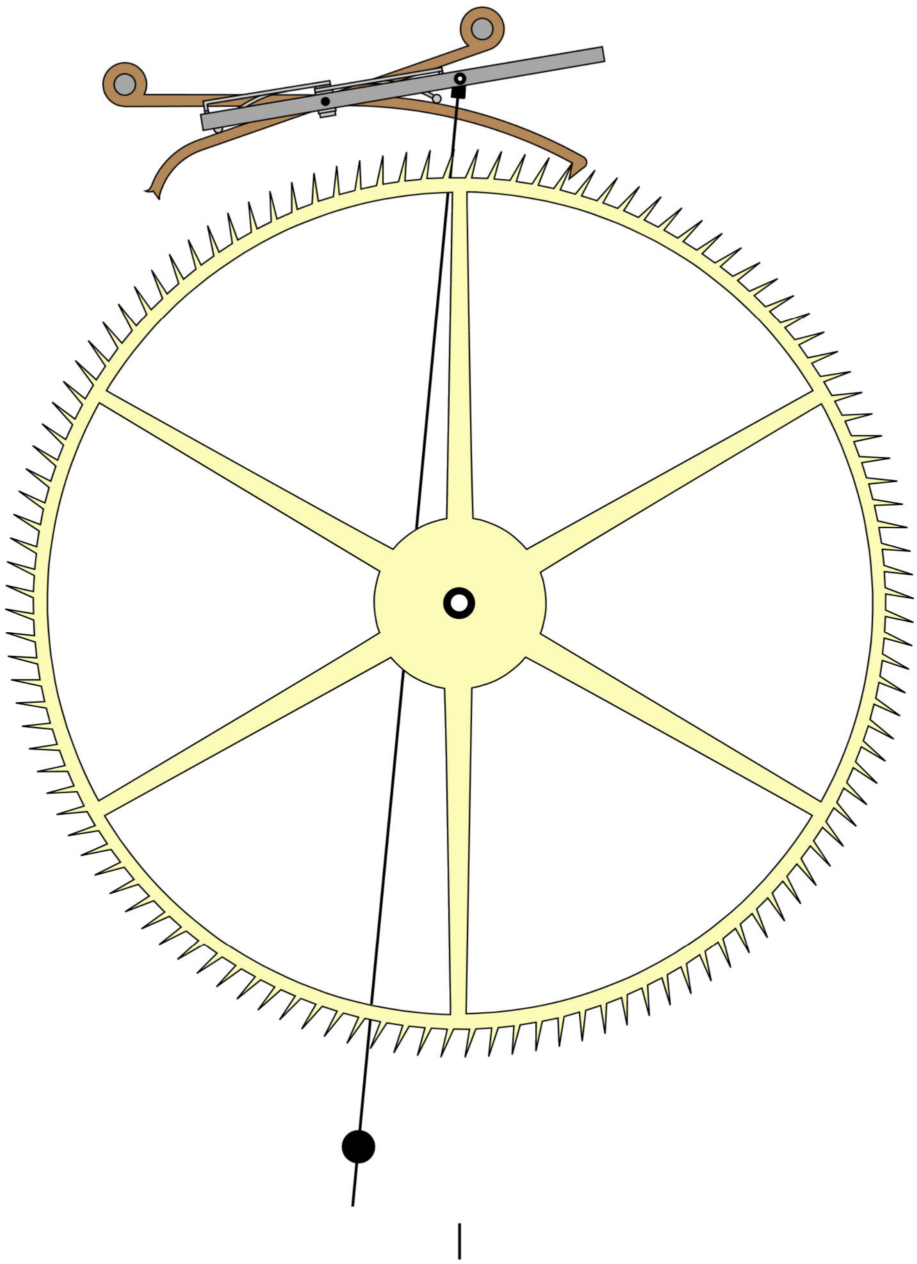


Figure 6.01 - The single pivot grasshopper escapement intended for Harrison's Final Regulator.

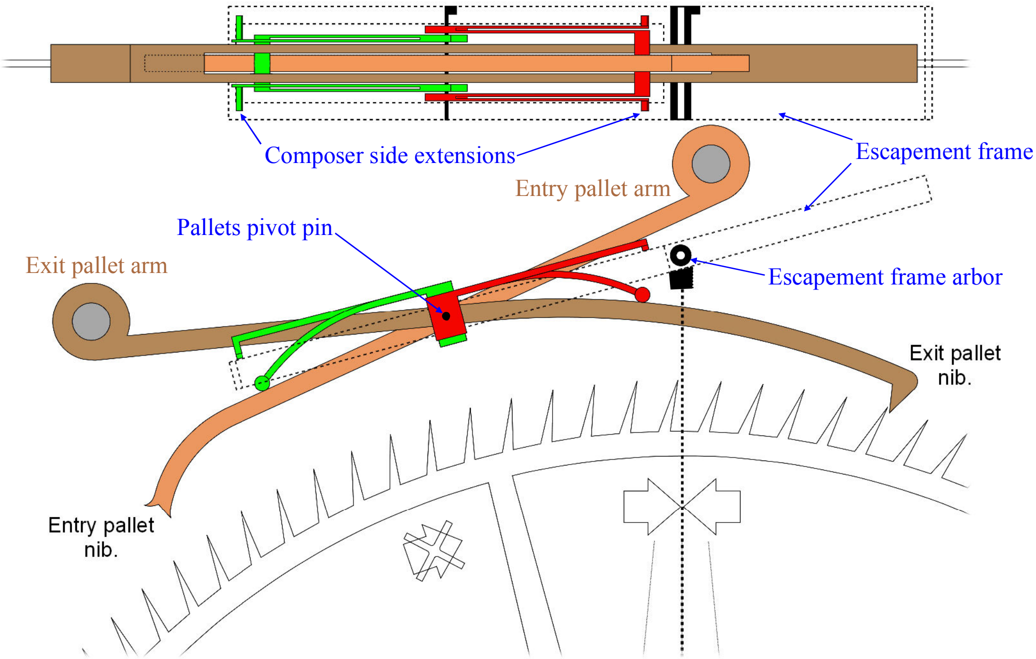


Figure 6.02 - Single pivot grasshopper escapement - mechanical arrangement

Figure 6.02 (above) - Represents the escapement in closer detail, viewed from above (the upper illustration) and from the front (the lower illustration), for a situation in which the pendulum is vertical and there is no torque to the escape wheel. A large part of the escape wheel is out of view (Figure 6.01 has already defined its complete form). Please make a mental note that, henceforth, all escape wheels should be assumed to rotate clockwise during normal operation, unless stated or indicated otherwise. Throughout this chapter the intention will be to allocate colours for the purpose of clear understanding, rather than to present true or elegant representations of material appearances. As an alternative to colour, one item in Fig.6.02, typically made of brass and called the **'escapement frame'**, is shown as completely transparent, with a broken outline. Such a method of representation serves to reveal the arrangement of other components otherwise obscured by the escapement frame. Observe that, using accepted terminology, the **'entry pallet'** is the first pallet to be met by any chosen escape wheel tooth as it 'enters' the escapement. By the same reasoning, the **'exit pallet'** is the last pallet encountered by any escape wheel tooth as it ‘exits’. Pallet **'nibs'** are formed at the ends of the **'pallet arms'**. The pallet arms are usually weighted at the **'tail'** ends, perhaps with circular inserts of lead, to such a degree that they are tail heavy. The arms are, therefore, constantly inclined to rotate clockwise (entry pallet arm) and anticlockwise (exit pallet arm) about a shared **‘pallets pivot pin’**, unless constrained to do otherwise. The red and green items are called **'composers'**. Their function will be described in detail very shortly.

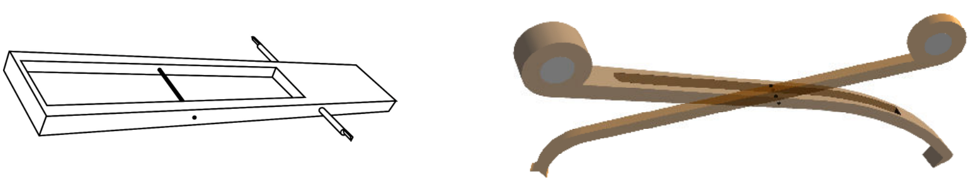


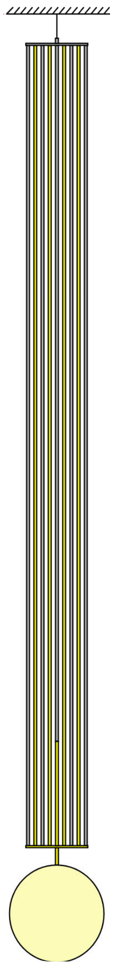
Figure 6.03 - Escapement frame and pallet arms.

Figure 6.03 (above) - The left hand illustration of Figure 6.03 shows the escapement frame in three dimensions to a reduced scale, in order to further clarify the mechanical arrangement. The right hand figure shows how the entry pallet arm pierces the exit pallet arm, whilst permitting independent, free rotation about a common, single pivot pin, probably of hard brass. The origins of the name **'single pivot'** will now be obvious. The pallet arms are usually created entirely from wood (subject to the selection of suitable types), for reasons to be explained later.

The single pivot for the pallet arms is affixed rigidly, without freedom to rotate or slide, within the escapement frame, which is, in turn, affixed to the pallet arbor, more appropriately referred to as the **'escapement frame arbor'**. The pendulum crutch is affixed, without freedom, to the escapement frame arbor and thereby transmits torque from the escapement to the pendulum Each end of the escapement frame arbor incorporates **'knife edge pivots'**, consisting of small, hardened steel V-shaped ends rocking upon a fine, sharp, axial V-shaped groove formed upon each of the upper surfaces of two glass plate pivot supports. Knife edge pivots are virtually friction free and require no lubrication. For simplicity and to avoid unnecessary clutter, the pendulum is represented by a single broken line and is shown as being attached without freedom to the escapement frame. In reality, Harrison's wonderful gridiron pendulum (loosely illustrated below) would be an almost obligatory fitment.

Also mounted freely and independently upon the single pallets pivot pin are two additional components, usually of brass, called **'composers'**, one for the entry side (coloured green) and one for the exit side (in red). Both composer are clearly nose heavy, the **'nose'** being the free end. Thus, the entry composer (green) is constantly inclined to rotate anticlockwise about the single pallets pivot pin and the exit composer (red) is constantly inclined to rotate clockwise about the same pin. The interaction between composers and pallet arms will be described in careful stages in the next section.

Note with particular care that each composer incorporates small extensions to either side. When those extensions to either composer contact the escapement frame (as is illustrated in figure 6.02), further downward movement of the nose of that composer is prevented.



Harrison's ingenious gridiron pendulum.

Typically a metre or thereabouts in overall physical length. Nine parallel rods (wires), four of steel, four of brass, one (central) of brass and steel joined by a 'tin whistle' compensation adjuster. Put simply, adjacent rods are joined rigidly at their ends such that, as atmospheric temperature increases (or decreases) the combined expansion (or contraction) of the brass rods is offset, to the necessary extent and to an adjustable degree, by the expansion (or contraction) of the steel and central brass/steel rods.

COMPONENTS AND THEIR BEHAVIOUR

There are six major escapement components, including the escape wheel, four of which share a common, single pivot (the 'pallets pivot pin'). Each of the four components sharing the pallets pivot may rotate about that pivot with complete independence, provided that they do not come into contact with any other component. However, at no time during normal escapement operation does independent behaviour completely occur, for there are constant relative motions and frequent interactions (which is why, amongst other things, the grasshopper escapement is so very fascinating to observe in action).

When simple contact between two components occurs, the more dominant component (which will be clearly identified in the course of any future descriptions) will thereafter dictate the motion of the other. The entire escapement sequence of operation is a fascinating, repeating series of such simple interactions. An unambiguous understanding merely requires a patient and careful inspection of every single, discrete interaction, presented in a logical sequence. That process, presented shortly, is extremely straightforward and easy to understand, if a little tedious for those who are quick to grasp such concepts. The firm objective (ambition) is to convey a complete, correct understanding to *any* reader, whatever their age (within reason), training, aptitudes or background.

This section presents the single pivot grasshopper escapement in reducing states of 'undress', beginning with the completely bare escapement frame and rigidly attached pendulum. Components will be added in a logical sequence, accompanied by explanations of the functioning of each. Without apology, there will be some repetition of explanations, in order to ensure that a crystal clear understanding and recall of the individual and combined behaviour of all components is attained.

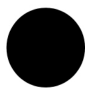
CONVENTIONS

Unless stated otherwise: all views are from the front of an imaginary clock movement, all escape wheels normally rotate clockwise and all dimensions are in millimetres (mm) and degrees (deg).

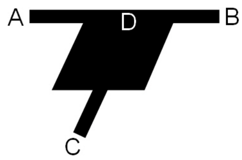
A key to the symbolic conventions used in the illustrations that follow is presented below:

'Anchored pivot' (greatly enlarged).
Rigidly incorporated into an immovable object. ‘Earthed’.

'Travelling pivot' (greatly enlarged).
Rigidly attached to a moving object. Moves in unison with that object.



'Rigid attachment' (greatly enlarged).
Prevents relative movement between attached objects
A, B and C rigidly connected at D.



Small, **symbolic pushing hands**, or a single hand, (greatly enlarged, right) indicate that a component is being held firmly in the position shown, or pushed in the direction shown, by an imaginary assistant. In order to avoid any possibility of misunderstanding, please note that the symbolic hands are, most emphatically, not to scale.

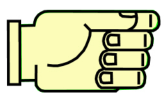


Figure 6.04 (next) - Presents a view from the front of the single pivot grasshopper escapement, with many of the components removed.

The **'escapement frame'** is shown edge-on (as a rectangle), pivoted about an anchored pivot. As described earlier, the anchored pivot is, in reality, the **'escapement frame arbor'** supported upon knife edges at each end.

The pendulum, or more precisely the upper part of the rod of the pendulum, is shown as a broken line, connected to the escapement frame by a rigid attachment. The pendulum is illustrated as being vertical in this instance. The opposing symbolic pushing hands either side of the pendulum indicate that it is being held firmly in the position shown by a willing assistant.

During normal escapement operation, the minimum pendulum displacement required for correct escapement functioning is defined by two markers, to the left and right of the vertical position, which are parts of lines radiating from the escapement frame arbor. In reality, the pendulum would be supplied with excess energy, in order to guarantee continuous operation whilst exposed to variable external influences. Those excesses of pendulum energy would result in pendulum displacements slightly beyond the two markers. The excess amplitude will be referred to (whether strictly correct or not) as **'overswing'**. A third line, somewhat obscured by the pendulum, but clearly visible in the next illustration (Fig. 6.05), coincides with the vertical position of the pendulum.

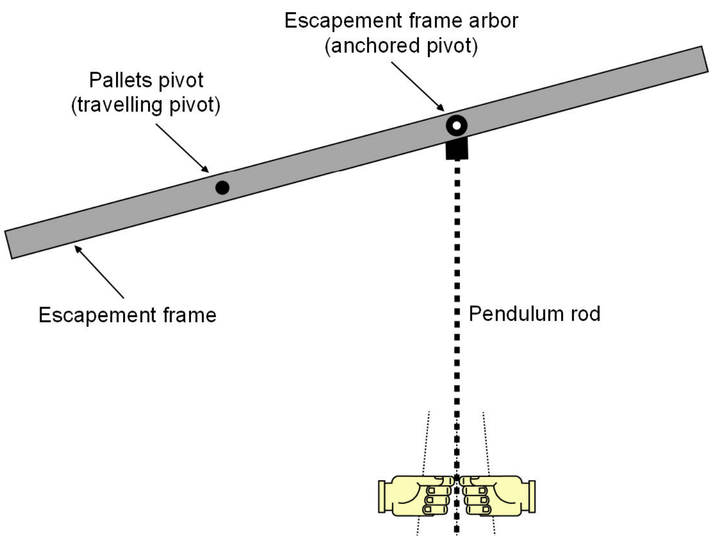


Figure 6.04 - Escapement frame, pendulum held in the vertical position.

Figure 6.05 (next) - Demonstrates the effect of releasing the hold on the pendulum and applying and maintaining a clockwise torque to the escapement frame arbor (anchored pivot), indicated by the symbolic pushing hand positioned (purely for convenience of illustration) near the pallets pivot. By virtue of the applied torque, the rigidly connected pendulum and escapement frame have been forced to rotate clockwise about the escapement frame arbor (anchored pivot). That displacement will be resisted by the Earth's gravity, acting upon their combined mass. When the applied torque and the effect of gravity are in balance, the pendulum will take up a stationary position to the left of vertical, as illustrated. The pendulum is shown aligned with the left-hand marker, for convenience of illustration and for no other reason. The third, central, marker is now clearly visible, aligned with a vertical through the escapement frame arbor (vertical pendulum position).

Observe that the **'pallets pivot'** (solid black circle) has been obliged to move in unison with the escapement frame: the pallets pivot is a 'travelling pivot'.

In normal operation, the torque required to maintain continuous motion of the pendulum need only be sufficient in magnitude to maintain the necessary amplitude, making up for any energy losses due to air resistance and flexing of the suspension spring etc. It should be bourne in mind, therefore, that typical operating torques and forces are significantly lower than those suggested by the illustrations.

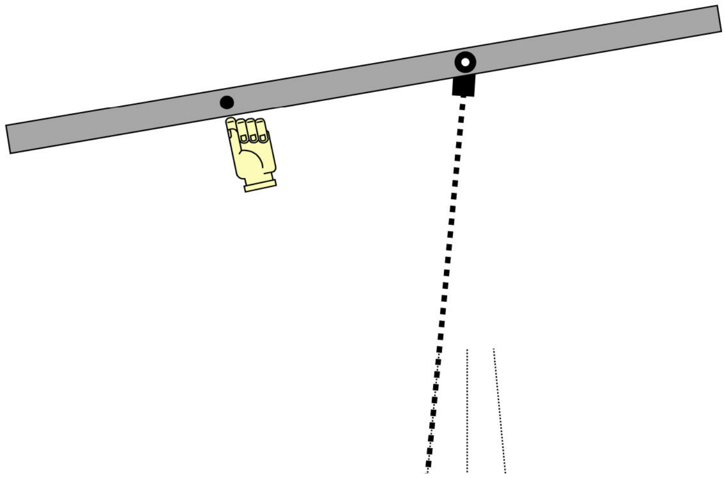


Figure 6.05 - Clockwise torque applied to the escapement frame arbor.

Figure 6.06 (next) - Reversing the situation depicted in Figure 6.05, an anticlockwise torque has been applied and maintained about the escapement frame arbor (anchored pivot), indicated by the symbolic pushing hand near the pallets pivot. The pendulum and escapement frame have rotated anticlockwise about the escapement frame arbor (anchored pivot) and the pendulum has swing to the right of vertical, until a balance of torques has been achieved.

Again, the pallets pivot has been obliged to move in unison with the escapement frame, as illustrated.

If a sequence consisting of Figures 6.05 and 6.06 was to be repeated in a continuous cycle, it would demonstrate how the motion of the pendulum could be sustained, by alternately applying sufficient clockwise and anticlockwise torque to the escapement frame arbor at precisely the required times.

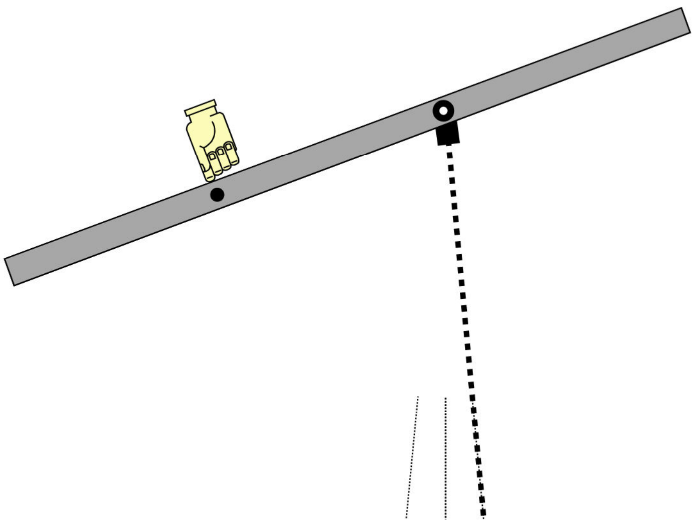


Figure 6.06 - Anticlockwise torque applied to the escapement frame arbor.

Figure 6.07 (next) - Two **'composers'** have been added to the previous configuration, the entry composer (green) being to the left and the exit composer (red) to the right of that. The escapement frame is shown as a transparent rectangle with a broken outline, in order to clarify the mechanical arrangement within. Although the pendulum is shown to be vertical, this is in no way relevant to the immediate explanation of composer operation. The pallets pivot pin is rigidly attached to the escapement frame and moves with the frame (it is a 'travelling pivot'). The composers share the pallets pivot pin and are free to rotate about it. **It is essential to understand that the composers are completely independent of each other. Any rotation of one composer about the pallets pivot pin will not affect the position or motion (if any) of the other composer.**

The composers are obviously nose heavy, the nose being furthest from the pivot. The entry composer is at all times attempting to rotate anticlockwise about the pallets pivot. The exit composer, by the same reasoning, will at all times be attempting to rotate clockwise about the pallets pivot.

Should the escapement frame rotate about the escapement frame arbor (anchored pivot), it is clear that the pallets pivot will be obliged to move in unison with it. In such a situation the pivot points of both composers will also be obliged to move with the pallets pivot.

Two broken circles highlight the points of contact between the **small side extensions (see fig. 6.2 should you seek a reminder)** incorporated into the composers and the escapement frame. In Figure 6.07, (next figure), the composer side extensions are visible as small square elements at the ends of the upper composer limbs, surrounded by and central to the broken highlighting circles. Any further rotation of the nose-heavy composers about the pallets pivot pin will be prevented by the side extensions and the composers will be obliged to rest in the positions shown. Any normal rotation of the escapement frame and rigidly connected pendulum will not affect the composer resting positions relative to the escapement frame - the entire assembly will simply rotate, completely unaltered, about the escapement frame arbor (anchored pivot).

In Figure 6.07 the composers are shown as green (entry composer) and red (exit composer`) in order to emphasise their unfamiliar forms and positions. In future illustrations, those colours will only be adopted when an applicable composer moves, relative to the escapement frame, from the ‘resting’ positions shown.

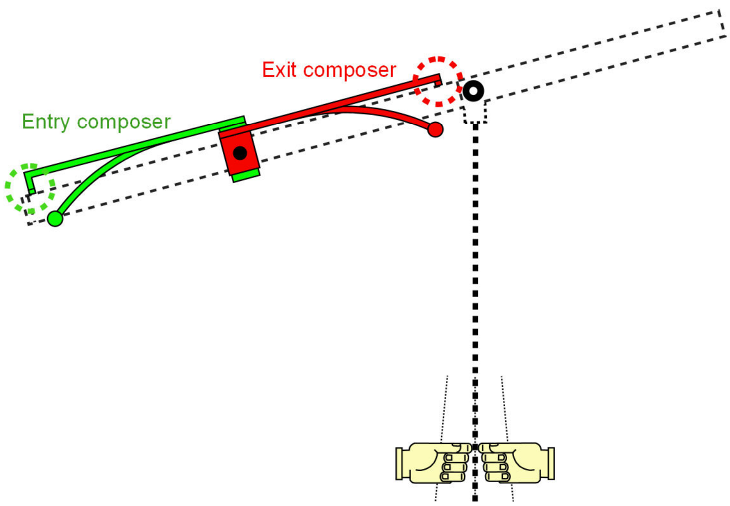


Figure 6.07 - Composers resting upon escapement frame, pendulum held in the vertical position.

Figure 6.08 (next) - The pendulum is being held in a vertical position and the noses of both composers have been raised by entirely separate, externally applied forces, indicated by the symbolic hands (our assistant apparently has four hands). In an effort to provide the clearest illustration, the amount of movement shown is considerably greater than would be experienced in normal escapement operation. The entry composer (green) side extensions have broken contact with the escapement frame and the composer is, therefore, free to generate continuous resistance to the applied force, in the form of an anticlockwise torque about the pallets pivot. The exit composer (red) has also broken contact with the escapement frame and is generating a completely independent, continuous resistance to the applied force, in the form of a clockwise torque about the pallets pivot.

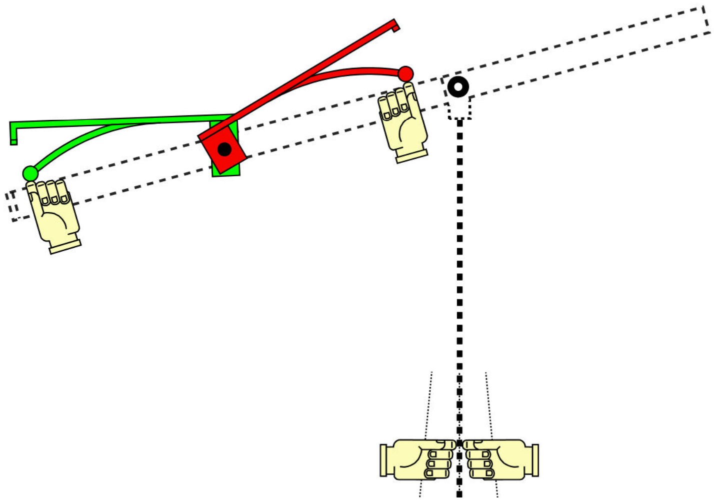


Figure 6.08 - Composers raised (exaggerated), pendulum held in the vertical position.

Figure 6.09 (next) - The two **'pallet arms'** have been added to the previous configuration, the entry pallet arm being the shorter and straighter of the two in this example. The active ends of the pallet arms are referred to as the **'nibs'**. The escapement frame assembly is now complete. The escape wheel has yet to be included.

As described in an earlier section, the pallet arms are weighted such that they are tail heavy. Thus, the entry pallet arm will be continuously generating a clockwise torque about the pallets pivot, which will hold it in continuous contact with the circular cross-section feature on the nose end of the lower limb of the entry composer. The clockwise pallet arm torque is arranged to be less than the anticlockwise torque of the entry composer, with the consequence that the pallet arm and composer pairing generates, overall, an anticlockwise torque. That torque will hold the entry composer and pallet arm pairing in continuous contact with the escapement frame, the points of contact being the small side extensions to the entry composer. The exit pallet arm and composer pair will mirror the behaviour of the entry pair.

As an aside, it is obvious that the mass of the pallet arms composers and pivot pin, plus the mass of the escapement frame to the left of the escapement frame arbor, will act about the arbor as an anticlockwise torque. The escapement frame to the right of the arbor will produce a clockwise torque and may be sized and/or weighted, in order to manipulate clockwise torque, perhaps in order to achieve perfect balance or apply bias, if deemed necessary

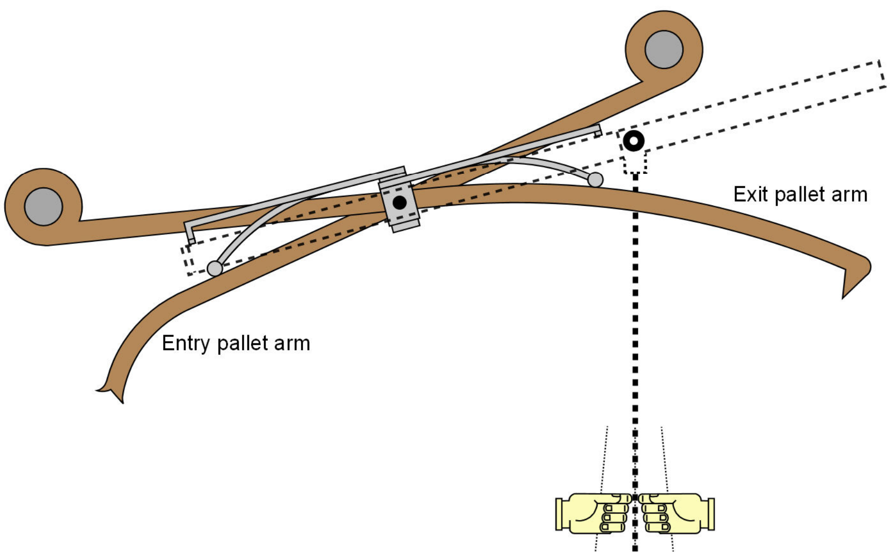


Figure 6.09 - Pallet arms resting upon composers, composers resting upon escapement frame.

Figure 6.10 (next) - The pendulum rod has been pushed and held to the left of vertical, obliging the rigidly attached escapement frame to rotate clockwise, through the same angle, about the escapement frame arbor (anchored pivot). The pallets pivot, composers and pallet arms must also rotate about the escapement frame arbor, again through the same angle. It is absolutely essential to note and memorise that there will be no rotation of any component about the pallets pivot. Thus, the small side extensions of the nose-heavy composers will continue to rest upon the escapement frame and the tail-heavy pallet arms will continue to rest in contact with their paired composers.

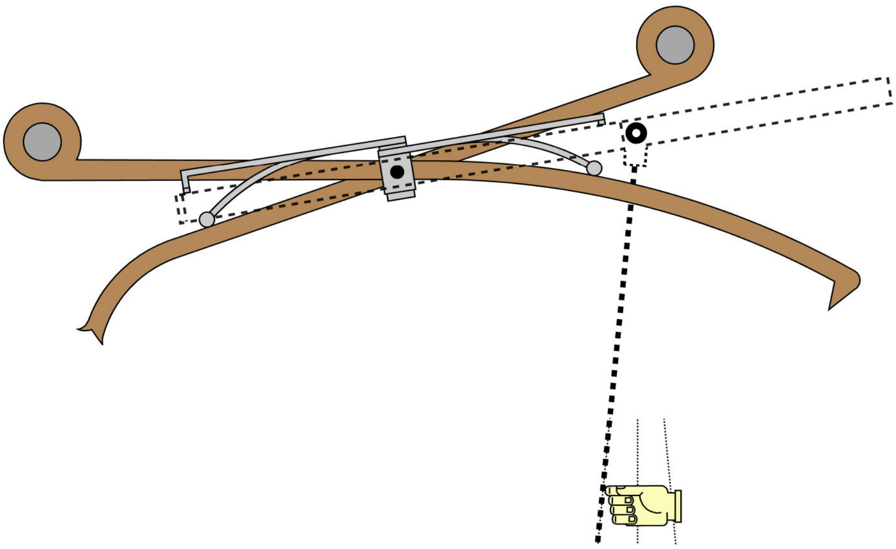


Figure 6.10 - Pendulum rod pushed to the left.

Figure 6.11 (next) - The pendulum rod has been pushed and held to the right of vertical. All components will rotate through the same angle about the escapement frame arbor (anchored pivot) without any motion relative to each other.

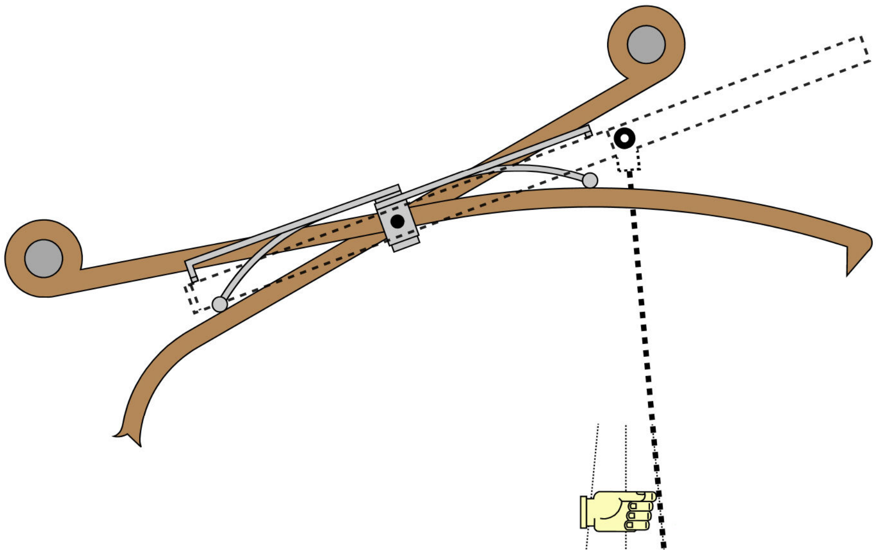


Figure 6.11 - Pendulum rod pushed to the right.

Figure 6.12 (next) - Illustrates the consequences of applying lift to the nib ends of the pallet arms. Considering only the entry side, sufficient lifting force will overcome the overall anticlockwise torque generated by the entry pallet arm and composer pairing. The broken green circle identifies that the small, square cross-section side extensions to the composer have been lifted away from the escapement frame, whilst the pallet arm and composer remain in contact with each other, by virtue of their opposing torques. The exit pallet arm and composer pair will mirror that situation.

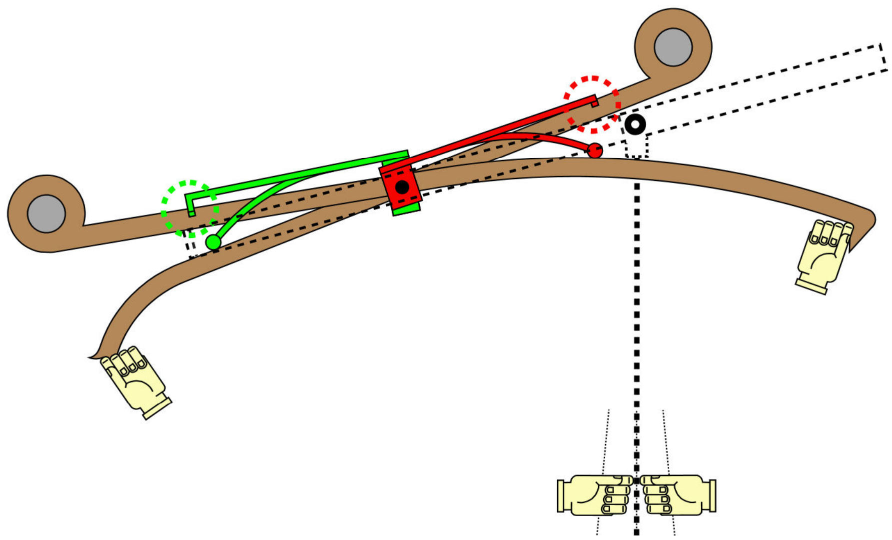


Figure 6.12 - Effects of raising the pallet arms at the nib ends.

Figure 6.13 (next) - Illustrates the consequences of holding the pendulum vertical and depressing the nib ends of the pallet arms. Considering only the entry pallet arm, sufficient downward force at the nib end will overcome the clockwise torque generated by the tail heaviness of the pallet arm about the pallets pivot pin. The broken green circle identifies that the entry pallet arm has been lowered out of contact with the composer, whilst the composer remains in contact with (resting upon) the escapement frame, by virtue of its inherent anticlockwise torque. The exit pallet arm and composer mirror that behaviour when a separate, downward force is applied to the nib end of the exit pallet.

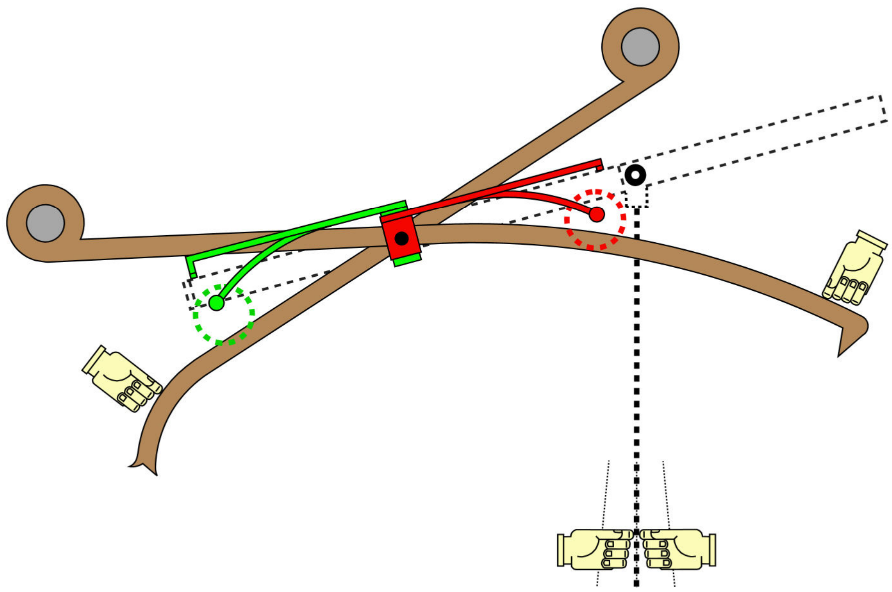


Figure 6.13 - Effects of depressing the pallet arms at the nib ends.

SETTING THE ESCAPEMENT IN MOTION

We are now equipped with sufficient understanding of component behaviour to be able to study the sequence of operations involved in setting the single pivot grasshopper escapement in motion, starting with an unwound clock, completely at rest. This study will serve, amongst more obvious things, to enhance our understanding of the escapement. The required sequence is, unsurprisingly, identical to part of the normal operating cycle, with the obvious exception that various manual inputs must be made at appropriate stages. The grasshopper demands particular care during this critical 'start-up' phase of operation, in order to avoid complete detachment of the pallets from the escape wheel (referred to as escapement **'trip'**), permitting hazardous, high speed, free-running of the escape wheel (referred to as escape wheel **'runaway'** and explained in a little more detail shortly). Component contact subsequent to runaway can result in extremely serious damage. There can be no doubt that by far the greatest historical cause of harm to grasshopper escapements has been a failure to observe the correct method of setting it in motion, leading to trip and runaway.

Figure 6.14 (next) - Represents the complete escapement, with the escapement frame now shown as an opaque grey object, rigidly connected to the pendulum. Below the escapement frame assembly lies the escape wheel, which normally rotates clockwise about an 'anchored pivot', out of view, at the centre of the escape wheel. The pendulum is being held in a vertical position by our willing assistant. The crossed arrow within the rim of the escape wheel indicates that the escape wheel is stationary and a note within the escape wheel states that there is no torque being applied to the escape wheel arbor.

Imperfect balancing of the escapement frame assembly will produce a torque which deflects the pendulum from the vertical. All illustrations assume perfect balancing and a vertical pendulum (when free).

Of considerable significance is the position of the exit pallet nib. The broken circle highlights both the nib and the closest escape wheel tooth anticlockwise removed from the nib. It should be clear that if a clockwise torque was applied to the escape wheel arbor, the escape wheel would rotate clockwise until the escape wheel tooth contacted the nib. Further escape wheel rotation would then be prevented. An **'impulse'** force would be created between the engaged escape wheel tooth tip and the exit pallet, which would translate into an anticlockwise torque about the escapement frame arbor. If the pendulum was free, it would be deflected and held to the right, to a degree that, for a given pendulum, would depend upon the magnitude of the torque being applied to the escape wheel arbor.

Such pallet engagement clearly relies upon there being sufficient length to the exit pallet nib, for if it were to be gradually shortened, a point would be reached where the escape wheel would be free to rotate unhindered. One purpose of the movement train is to increase the rate of rotation of the escape wheel relative to the weight barrel. A completely free escape wheel would, therefore, rotate at an ever increasing and eventually alarming rate. A 'runaway' situation is dangerous for both the clock and nearby operatives, since it would certainly culminate in the rapidly falling weight crashing to the bottom of the case, with possible damage. Not least, any contact between the speeding escape wheel and an escapement pallet would almost certainly result in the violent destruction of those components. There is, therefore, a clear incentive to ensure that the exit pallet nib is as long as other demands (such as correct and reliable escapement functioning), will permit. As illustrated, the pallet nib is of sufficient length to provide somewhat precarious protection against escapement trip.

A second possibility must also be considered. Beginning with the situation shown in **Figure 6.14 (next)**, should the pendulum be displaced to the right, the entire escapement frame assembly (frame, pallet arms, composers and pivot pin) would rotate (without any motion of any components relative to each other) anticlockwise about the escapement frame arbor, thereby rotating the exit pallet nib away from the escape wheel. Regardless of the length of the exit pallet nib, a point would eventually be reached where the nib would be lifted completely clear of the circular path of the escape wheel teeth tips and the escape wheel would be free to rotate unhindered, should torque be applied to its arbor. There is, therefore, a clear incentive to ensure that the pendulum is vertical and stationary, before torque is applied to the escape wheel arbor. In practice, in view of the often precarious protection offered by the exit pallet nib, there is a further precaution that should be taken before applying torque to the escape wheel arbor. All will be revealed in good time, at the appropriate point.

Observe that, in comparison with the exit pallet, the entry pallet offers almost no assistance in the matter of preventing or minimising the risk of escapement trip. The resting position of the entry pallet nib, some distance from the escape wheel, is totally unsuited to such a purpose.

As will be demonstrated during the course of this publication, there can be little doubt that the grasshopper escapement is superior to most, if not all, other purely mechanical forms. However, previous warnings serve to highlight one major weakness: incorrect construction and/or careless or ignorant handling can be responsible for the cause of escapement trip and high speed escape wheel runaway, leading to severe damage. As mentioned earlier, no doubt as a consequence of the latter deficiency, few original Harrison pallet arms have survived to this day. Furthermore, it is conceivable that generations of horologists have unfairly interpreted a susceptibility to careless handling as sufficient reason to avoid the grasshopper, in favour of escapements of significantly poorer performance.

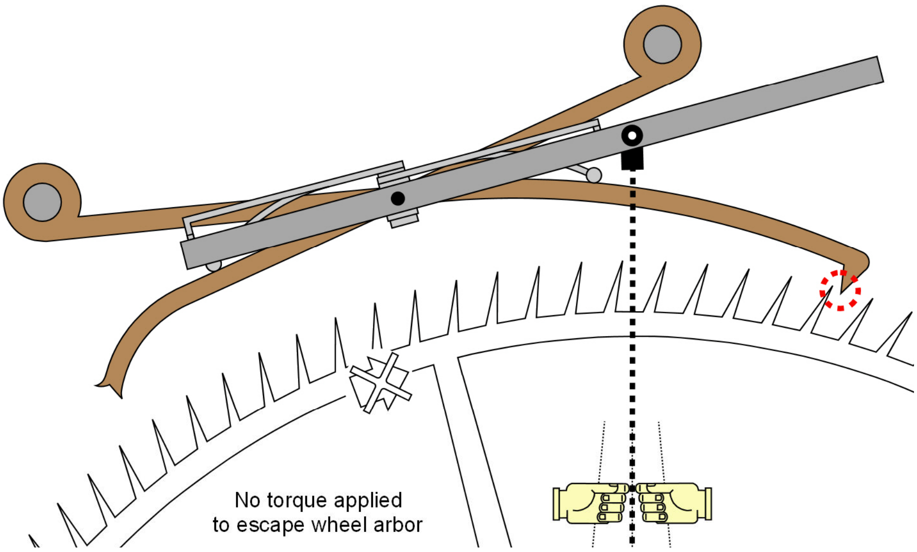


Figure 6.14 - Escapement and escape wheel at rest, pendulum vertical.

Figure 6.15 (next) - The 'at rest' scenario of Figure 6.14 is repeated, with the exception that the exit pallet arm has been *gently* depressed at the nib end, by our helpful assistant, until the underside contacts the escape wheel, as highlighted by the broken circle. As the exit pallet arm is lowered, it will separate from the exit composer, which is held back by the escapement frame. The escape wheel tooth in contact with the exit pallet is shown with a dark infill, which will ease the process of understanding the remaining sequence and eventual outcome.

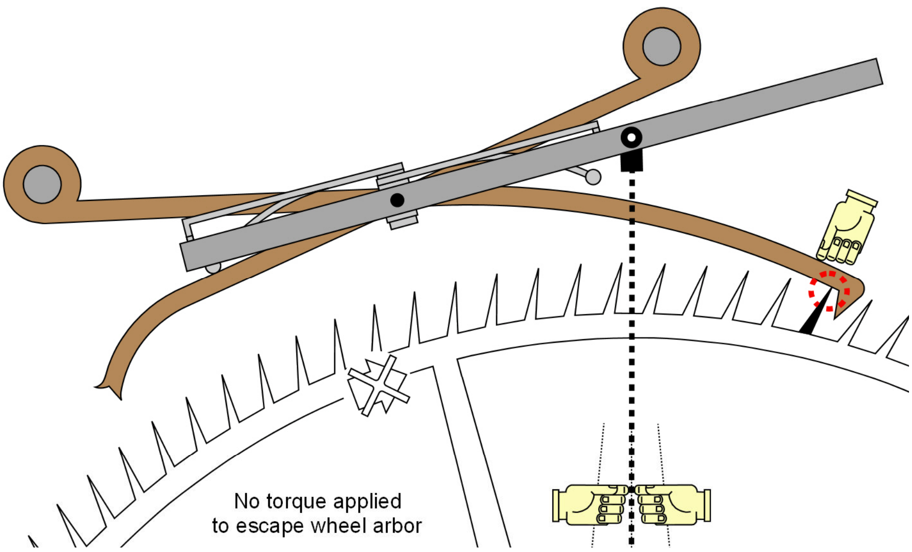


Figure 6.15 - Exit pallet arm depressed onto an escape wheel tooth tip.

Figure 6.16 (next) - Torque ('turning effort') has been applied to the escape wheel, by winding the clock, thereby raising the driving weight and supplying energy to the movement. Provided that the manually applied depression at the exit pallet nib is *only just* sufficient to overcome the tail-heaviness of the pallet arm, the momentary effect of winding the clock will be that the dark escape wheel tooth tip in contact with the exit pallet arm will slide along the underside of the arm until it reaches the corner formed between the underside and the pallet nib. The corner in question is highlighted by the broken red circle. Care is required, for excessive depression at the exit pallet nib would prevent such a sliding action, by generating more friction than the escape wheel can overcome.

Further escape wheel rotation is now prevented, by virtue of the dark engaged escape wheel tooth tip being restrained, albeit in a clockwise direction only, in the corner of the nib. That corner will henceforth be referred to as the '**locking corner'**, in this case for the exit pallet. The arrow within the rim of the escape wheel is crossed, indicating that the escape wheel is stationary, despite the continuous application of torque from the train.

If sufficient torque has been applied to the escape wheel and provided that it is maintained, we may remove the manually applied force depressing the exit pallet nib onto the escape wheel (symbolic pushing hand removed). Static friction between the dark escape wheel tooth tip and the exit pallet locking corner would, if of sufficient magnitude, hold the exit pallet nib in position, overcoming the tail-heaviness of the pallet arm. This situation will be referred to henceforth as **'capture'**, the pallet locking corner having been 'captured' by friction at the engaged escape wheel tooth tip. We may now appreciate one ingenious reason for using a suitable wood as a pallet arm material. Many other materials, such as metal, would generate lower static friction, offering less secure capture.

The situation of Figure 6.16 is more secure than that that of Figure 6.14. Nevertheless, in view of the aforementioned disastrous consequences of escapement trip and escape wheel runaway, a cautious operator might consider it desirable to maintain gentle depression of the exit pallet nib onto the escape wheel, in order to guarantee the most secure escape wheel restraint. That optional precaution will not be illustrated further, in order to avoid confusion, although it will be acknowledged when the release of manual input is required.

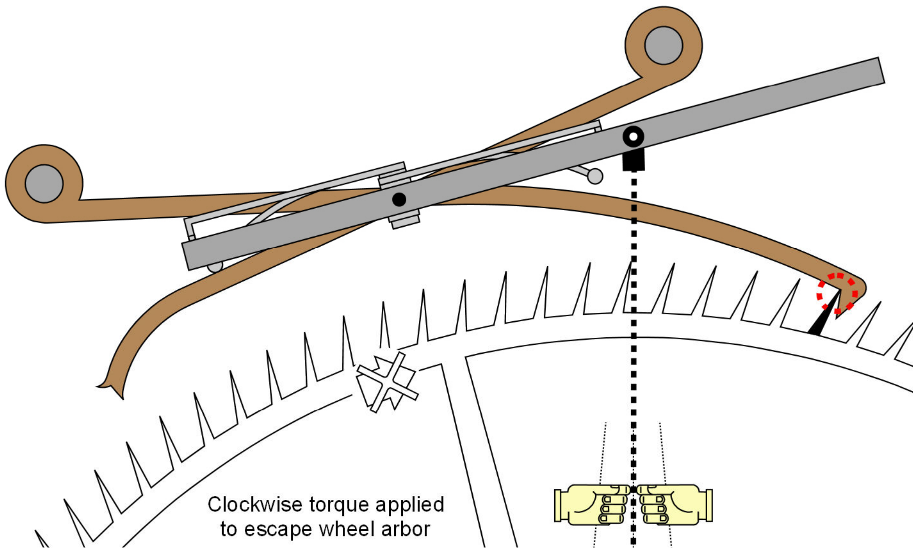


Figure 6.16 - Torque applied by the driving weight, via the movement train, to the escape wheel. Exit pallet locking corner captured by the escape wheel.

Figure 6.17 (next) - Our assistant is beginning to apply a force to the pendulum, to the right, as indicated by the symbolic pushing hand acting upon the pendulum rod. Although this figure is apparently identical to Figure 6.16, with the exception of differing manual inputs and movement symbols, it is intended to represent a point at which various motions are beginning to occur. As a result of the applied force, the escapement frame will begin to rotate anticlockwise about the escapement frame arbor, as indicated by the grey arrow near the pallets pivot.

The exit pallet locking corner will remain captured by the engaged dark escape wheel tooth (continued capture being indicated by the unbroken red circle at the exit pallet locking corner). The pallets pivot pin will begin to move closer to the escape wheel, since it is rotating in unison with the escapement frame, about the escapement frame arbor. That movement of the pallets pivot pin will oblige the exit pallet arm to begin to rotate clockwise about the pallets pivot pin (I repeat - the *pallets pivot pin*) as the escape wheel begins to rotate clockwise.

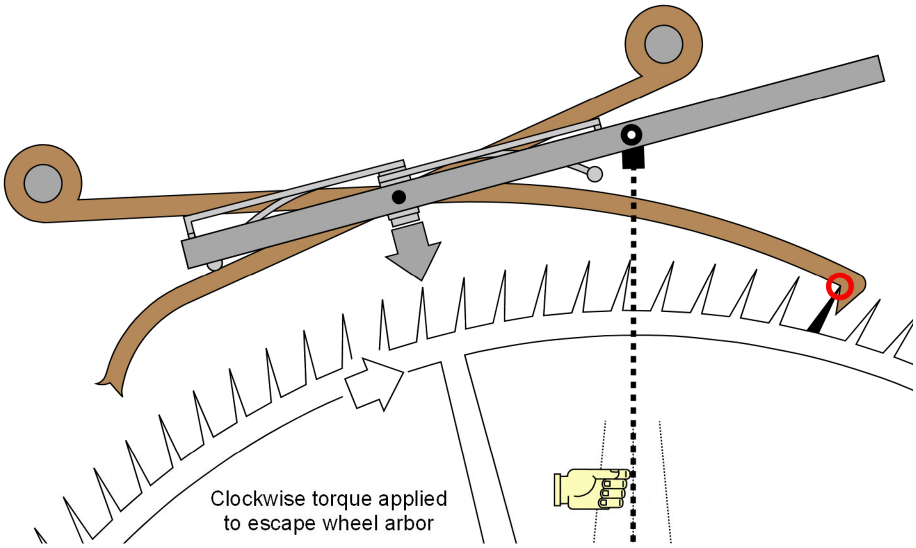


Figure 6.17 - Commencement of manual pendulum displacement to the right. Exit pallet impulsing.

Figure 6.18 (next) and Figure 6.19 (next but one) - These two figures represent two events occurring virtually simultaneously. However, the process of complete understanding requires that they be described and understood separately.

Figure 6.18 (next) - Our assistant has continued to push the pendulum to the right, rotating the escapement frame further anticlockwise. The pallets pivot pin, entry pallet arm and both composers rotate in unison with the escapement frame about the escapement frame arbor, but the exit pallet arm, still captured at its locking corner by the escape wheel, is obliged to rotate further clockwise about the pallets pivot pin.

Eventually, as shown, the entry pallet will contact the escape wheel, the geometry being designed to ensure that contact between the pallet nib and escape wheel tooth tip occurs at precisely the entry pallet locking corner. There may be an audible 'click' on contact, depending upon the exuberance of our assistant. The entry pallet will immediately prevent further clockwise rotation of the escape wheel, as indicated by the crossed arrow and will be 'captured' the newly engaged escape wheel tooth tip.

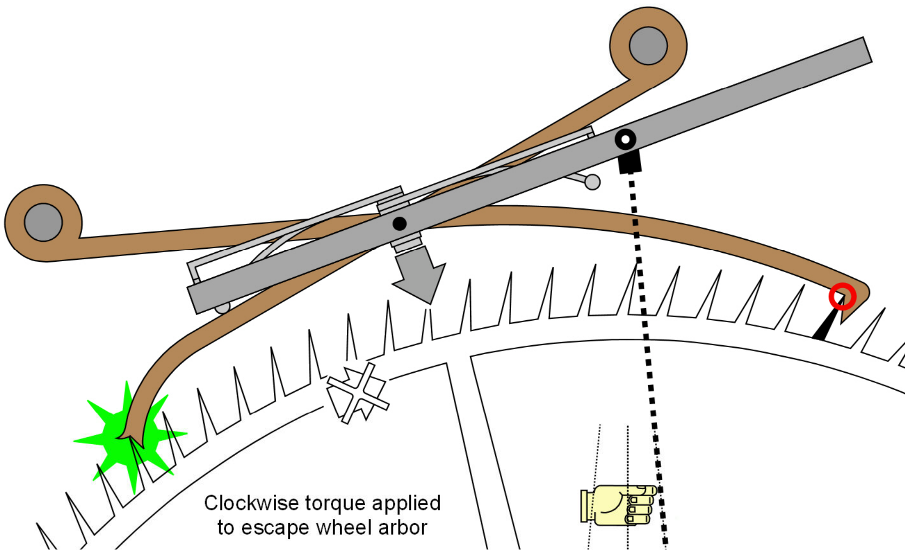


Figure 6.18 - Entry pallet locking corner captured by the escape wheel.

One reason for choosing the exit pallet arm for first engagement with the escape wheel, in preference to the entry pallet arm, should now be clear. The underside of the exit pallet arm provides a convenient physical guide towards the locking corner, whereas the entry pallet locking corner, formed by the inverted 'V' at the nib end, would require precise placement on the tip of an escape wheel tooth and would be somewhat more difficult to hold in that position, if so desired. Bear in mind that the escapement is often positioned within a movement, which is itself housed inside a display case, often rendering access difficult. Any technique offering assistance should, therefore, be accepted.

Figure 6.19 (next) - The pendulum has been pushed further to the right by an imperceptible amount. The escapement frame has rotated further anticlockwise, thereby moving the pallets pivot pin closer to the escape wheel and obliging the entry pallet arm to rotate further clockwise about the pallets pivot. The only means by which the escapement can absorb such movements is to force the escape wheel into recoil, as indicated by the arrow within the escape wheel rim. As another consequence of the clockwise rotation of the entry pallet arm, the entry composer becomes imperceptibly detached from the escapement frame (and its illustrated colour changes from grey to green).

At the instant escape wheel recoil begins, capturing friction at the exit pallet is lost. The exit pallet arm, being tail heavy and detached from its composer, is free to rotate anticlockwise, away from the escape wheel. That event represents another of the three phases through which the grasshopper escapement cycles. It will be referred to herein as **'release'**, during which the applicable pallet locking corner is 'released' from the escape wheel tooth tip that previously held it captive. Observe that capture of the entry pallet has been responsible for the release of the exit pallet and that the time interval between entry pallet capture and exit pallet release is extremely short, to the extent that, for all practical purposes, the two events occur simultaneously.

The cautious operator, who has been maintaining pressure to the exit pallet nib as an additional precaution against trip, should release that pressure just before the entry pallet locking corner makes contact with the escape wheel, in order to permit unhindered release of the exit pallet. A failure to achieve that action before the required instant may disrupt the starting process and/or generate undesirable, possibly damaging, conflicting loads.

Shortly after exit pallet release, the exit pallet arm will contact the exit composer. Subject to the degree of tail weighting of the exit pallet arm, the motion and contact could be slow and gentle, generating almost no sound, or it could be rapid and hard, producing a clearly audible 'click'. There may even be some bounce of the pallet arm upon contact, creating a series of 'clicks' of diminishing volume. In extreme cases, contact could also induce bounce of the composer on the escapement frame, creating further sounds. Note carefully that, unlike the anchor and dead beat escapements, such sounds do not correspond to contact between an escape wheel tooth tip and a pallet locking corner and would, therefore, be of no assistance when setting the clock in beat.

The newly captured entry pallet is now receiving impulse from the escape wheel. That impulse acts along a line joining the entry pallet locking corner to the pallets pivot pin, which creates a clockwise torque about the escapement frame arbor, attempting to swing the pendulum from right to left. As a result, our assistant must expend a little more effort, in order to maintain the illustrated pendulum position.

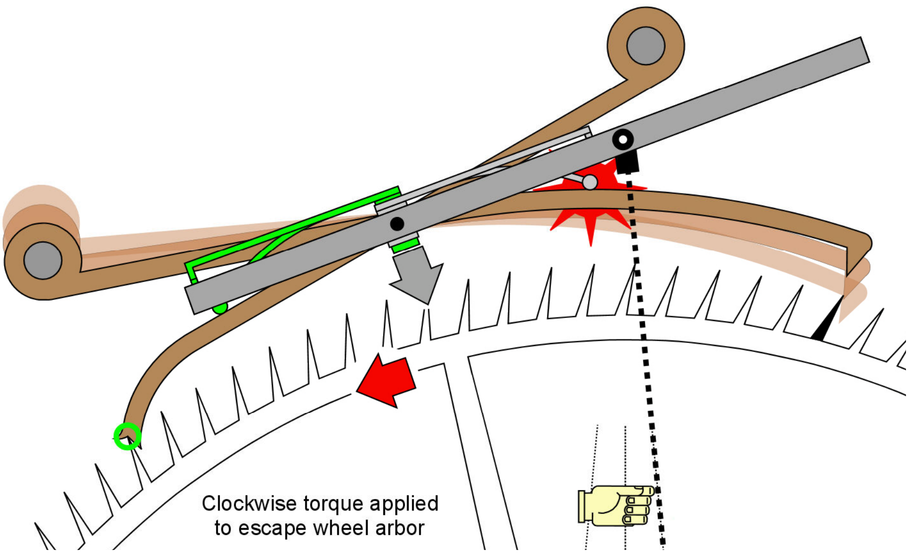


Figure 6.19 - Virtually coincident with Figure 6.18.
Entry composer begins to lift. Escape wheel recoil begins. Exit pallet released.

Figure 6.20 (next) - The pendulum displacement to the right is continued, slightly beyond the right hand displacement marker, in order to add a little more energy. That addition of energy ensures that the escapement will function reliably whilst subjected to variations in escape wheel torque and pendulum resistance. In normal operation, excess energy is achieved by setting the driving weight such that the pendulum always operates slightly beyond the minimum amplitude required for pallet capture and release. It will be necessary to adopt a 'trial-and-error' approach to setting the weight. Although **'supplementary arc'** is the correct term, it will be usefully descriptive, for those who are unfamiliar, to refer to the safe excess of pendulum amplitude as **'overswing'**. Overswing is the last of the three phases of operation of the grasshopper escapement. In comparison with the pendulum displacement of previous stages, the additional pendulum amplitude during overswing is quite small, although its magnitude for reliable escapement operation is a function of geometry, quality of construction and care taken in adjustment. In normal operation, overswing occurs as the angular momentum of the pendulum is gradually overcome by the opposing torque generated by a newly captured pallet.

Returning to Figure 6.20, during overswing, the entry pallet arm is forced to rotate further clockwise about the pallets pivot pin and the escape wheel is, therefore, obliged to recoil. Another, significant outcome is that the entry pallet arm will lift the entry composer away from the escapement frame (composer changes from grey to green). By those means, excess motions during overswing are absorbed.

The start up sequence has now been completed.

The sequence from Figure 6.15 up to and including 6.20 illustrates one secure method of starting a timepiece fitted with a single pivot Harrison grasshopper escapement. Variations in approach may be possible, although any alternative methods must ensure that escapement trip and escape wheel runaway are prevented. It will now be appreciated that the starting of a stationary clock fitted with a grasshopper escapement demands care and attention. The common practice of winding without preparation, followed by indiscriminate swinging the pendulum will lead to certain disaster.

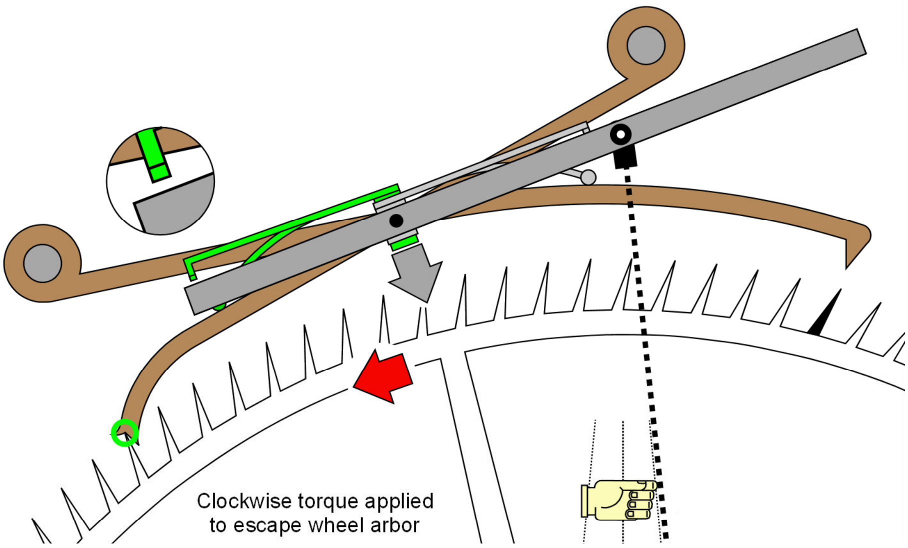


Figure 6.20 - Pendulum overswing. Escape wheel recoil continues. Entry pallet still captured. Entry composer visibly breaks contact with escapement frame.

CYCLE OF OPERATION

We shall now investigate the complete cycle of operation of the single pivot grasshopper escapement. It will be convenient to begin the cycle a brief instant after the situation depicted in Figure 6.20. In order to start that sequence, our helpful assistant must release the pendulum, which is being held, stationary, at the extremity of overswing to the right. From this point onwards, provided that the driving weight is adequate, the escapement will provide sufficient energy to maintain a continuous cycle of operation without further external assistance.

Figure 6.21 (next) - Gravity is accelerating the pendulum to the left and the rigidly attached escapement frame is rotating clockwise about the escapement frame arbor (anchored pivot). The captured entry pallet nib is receiving impulse from the escape wheel and the entry pallet arm is rotating anticlockwise about the pallets pivot. Impulse to the entry pallet nib acts along a line joining the pallet locking corner to the pallets pivot pin, which creates a clockwise torque about the escapement frame arbor, assisting gravity in accelerating the pendulum to the left. The entry composer (in unison with the entry pallet arm) rotates anticlockwise about the pallets pivot, moving its nose closer to the escapement frame. Note carefully the position of the dark escape wheel tooth closest to the exit pallet nib. That tooth will serve to record the progress of the escape wheel during the complete cycle of events, starting at the illustrated position.

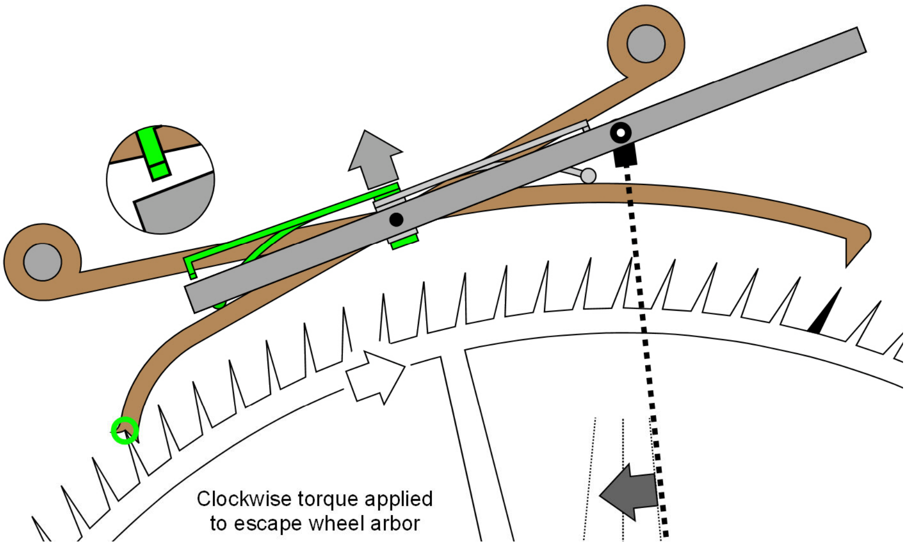


Figure 6.21 - Limit of overswing. Start of entry pallet impulse.

Figure 6.22 (next) - Overswing has just ended and the pendulum is, therefore, passing through the right hand amplitude marker. The entry pallet is still receiving impulse from the escape wheel, assisting gravity in swinging the pendulum from right to left. The anticlockwise rotations of the entry pallet arm and entry composer about the pallets pivot have only just reached the stage at which the composer has contacted the escapement frame (illustrated colour changes from green to grey). The point of contact between the composer and frame is highlighted by the broken green circle. Sound is rarely generated by this type of composer contact, the motion being a 'placement', rather than an impact.

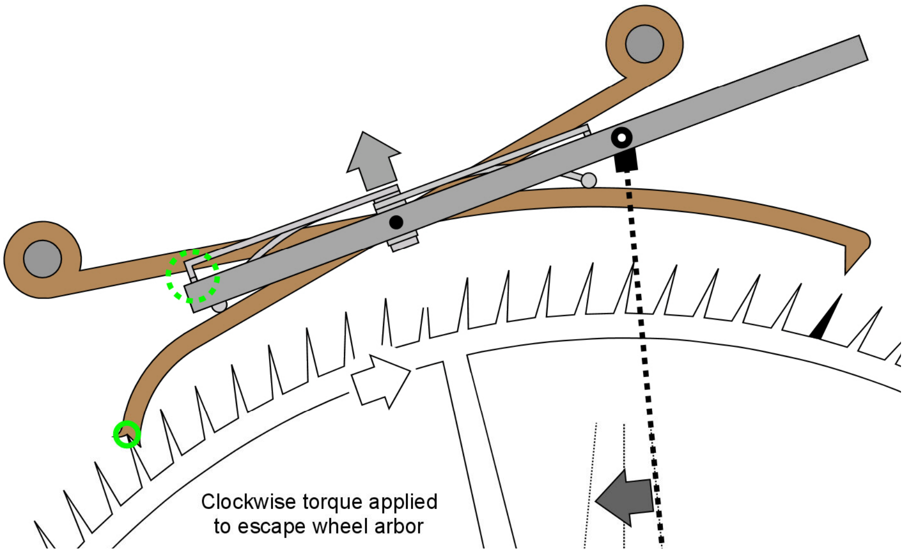


Figure 6.22 - Entry pallet impulsing. Entry composer halted by escapement frame.

Figure 6.23 (next) - As soon as the pendulum swings to the left of the position shown in the previous figure, the continued anticlockwise rotation of the entry pallet about the pallets pivot will lead to a loss of contact with the entry composer, which is left behind, resting upon the escapement frame. As illustrated, the pendulum is passing through mid swing and the entry pallet arm has rotated away from the composer to an obvious degree, as highlighted by the broken green circle.

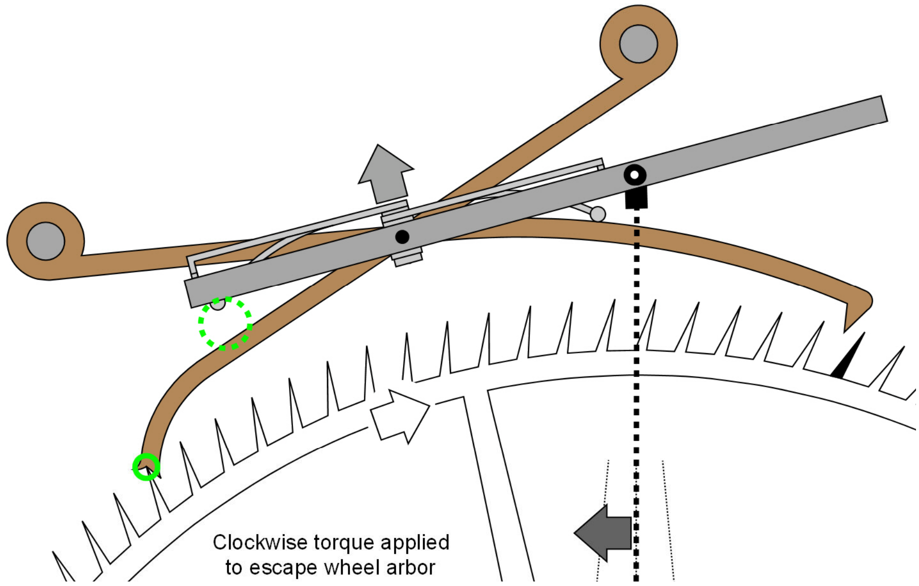


Figure 6.23 - Entry pallet impulsing. Pallet arm detached from composer.

Figure 6.24 (next) and Figure 6.25 (next but one) - These figures represent two events occurring virtually simultaneously. They will be described separately.

Figure 6.24 (next) - The exit pallet arm and exit composer pairing have rotated in unison with the escapement frame, about the escapement frame arbor. The exit pallet arm has encountered the escape wheel, the geometry having been arranged such that contact between the pallet nib and engaging escape wheel tooth tip occurs at precisely the exit pallet locking corner. A 'click' sound may be generated. The escape wheel is momentarily halted and the exit pallet locking corner is captured.

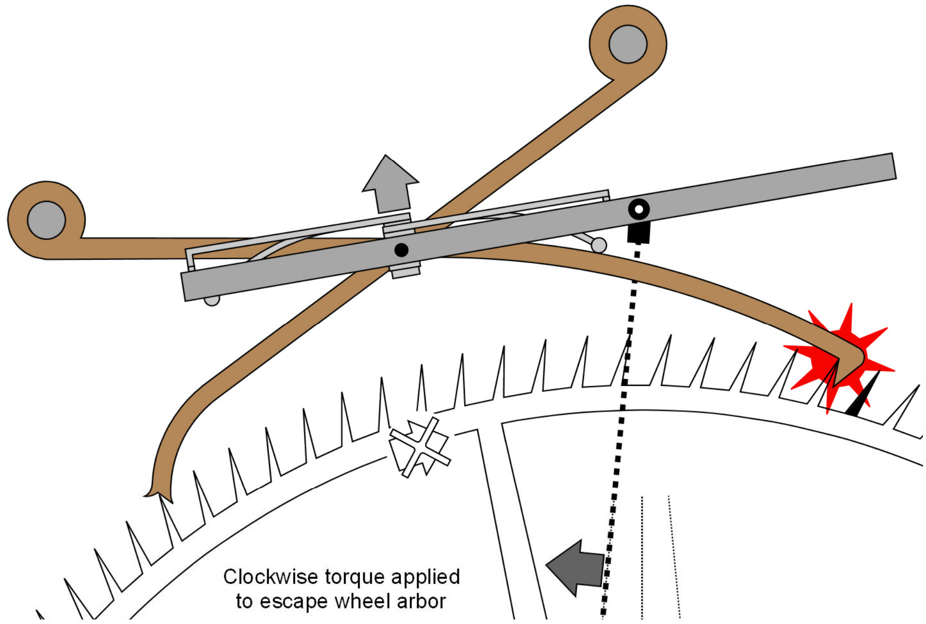


Figure 6.24 - Exit pallet capture. End of entry pallet impulse. Escape wheel halted.

Figure 6.25 (next) - Escape wheel impulse to the exit pallet applies anticlockwise torque to the escapement frame arbor. Diminishing pendulum momentum opposes that torque during overswing to the left. The escape wheel is recoiled by the exit pallet, which releases the entry pallet. The entry pallet rotates clockwise about the pallets pivot until arrested by the entry composer. There may be an audible 'click' as the entry pallet arm and composer meet. There may be further clicks if there is bounce. The exit pallet arm and composer rotate anticlockwise about the pallets pivot pin (illustrated exit composer changes to red).

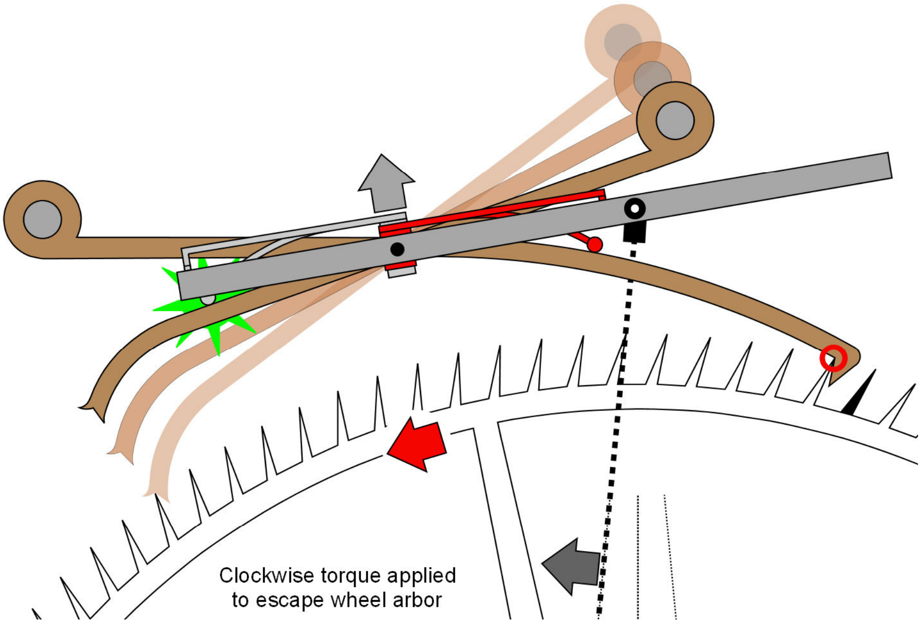


Figure 6.25 - Virtually coincident with Figure 6.24.
Escape wheel recoil and overswing begins. Entry pallet released. Exit composer begins to lift.

Figure 6.26 (next) - Pendulum momentum opposes exit pallet impulse. The pendulum, escapement frame and escape wheel will eventually stop, momentarily, at the limit of overswing to the left. The captured exit pallet arm has rotated the exit composer (now red) away from the escapement frame, as emphasised in the magnified view.

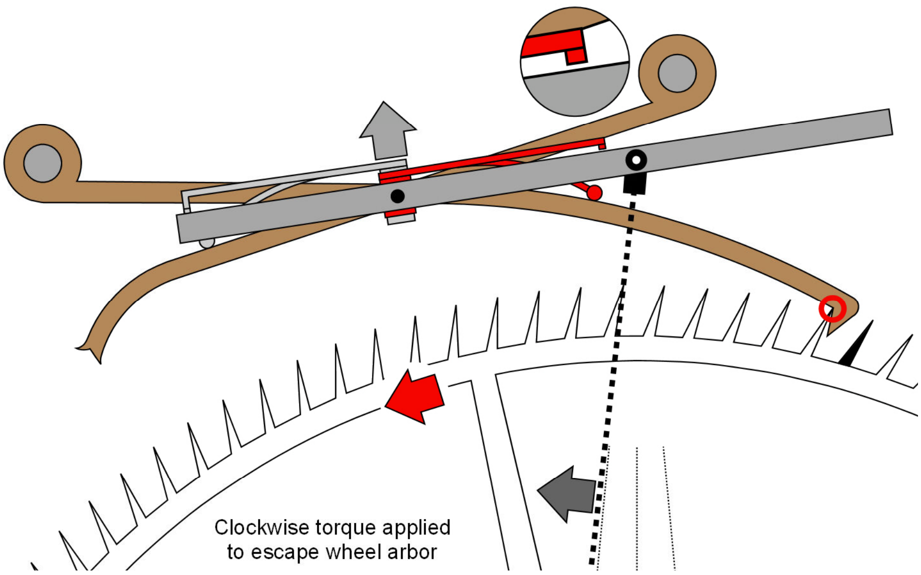


Figure 6.26 - Pendulum overswing. Escape wheel recoil continues. Exit pallet remains captured.
Exit composer visibly breaks contact with escapement frame.

Figure 6.27 (next) - The exit pallet is receiving impulse from the escape wheel and begins to rotate clockwise about the pallets pivot. The pendulum begins to swing to the right and the rigidly attached escapement frame rotates anticlockwise about the escapement frame arbor. The exit composer nose, still clear of the escapement frame, begins to rotate clockwise, with its paired exit pallet arm, about the pallets pivot.

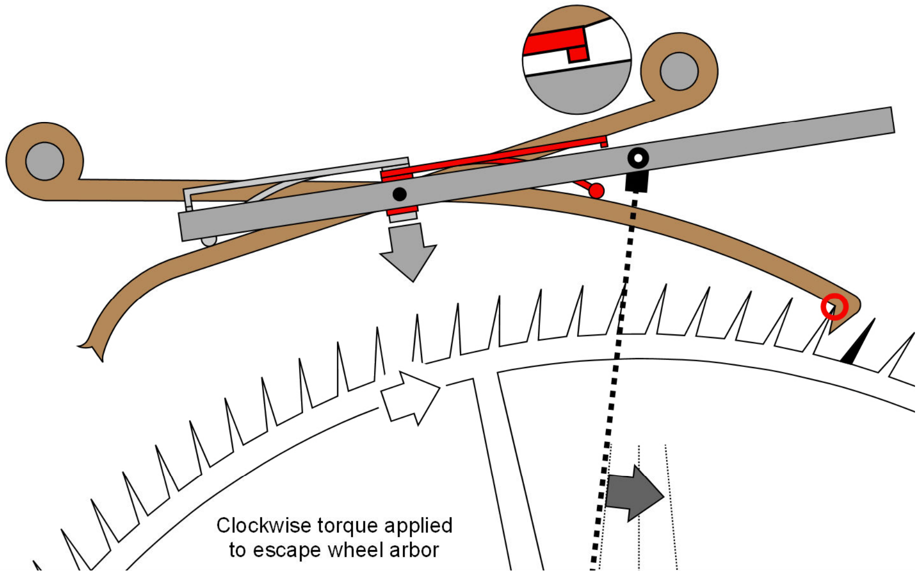


Figure 6.27 - Limit of overswing. Start of exit pallet impulse.

Figure 6.28 (next) - The pendulum has reached the end of overswing and is passing through the left hand amplitude marker. The exit pallet is still receiving impulse from the escape wheel, thereby assisting gravity in swinging the pendulum from left to right. The clockwise rotations of the exit pallet arm and entry composer about the pallets pivot have only just reached the point at which the composer has contacted the escapement frame (illustrated colour changes from red to grey). The point of contact between the composer and frame is highlighted by the broken red circle.

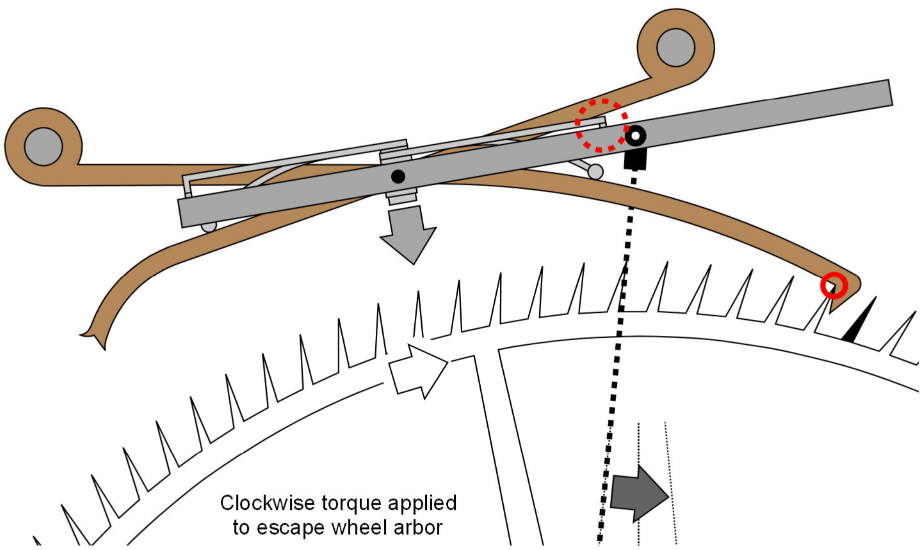


Figure 6.28 - Exit pallet impulsing. Exit composer halted by escapement frame.

Figure 6.29 (next) - The exit pallet continues to rotate clockwise about the pallets pivot pin, causing it to break contact with the exit composer (broken red circle), which is left behind, resting upon the escapement frame. As illustrated, the pendulum is passing through mid-swing and the exit pallet arm is well clear of the composer.

Note the position of the entry composer nib, compared to the previous figure. It is being rotated about the escapement frame arbor towards the escape wheel.

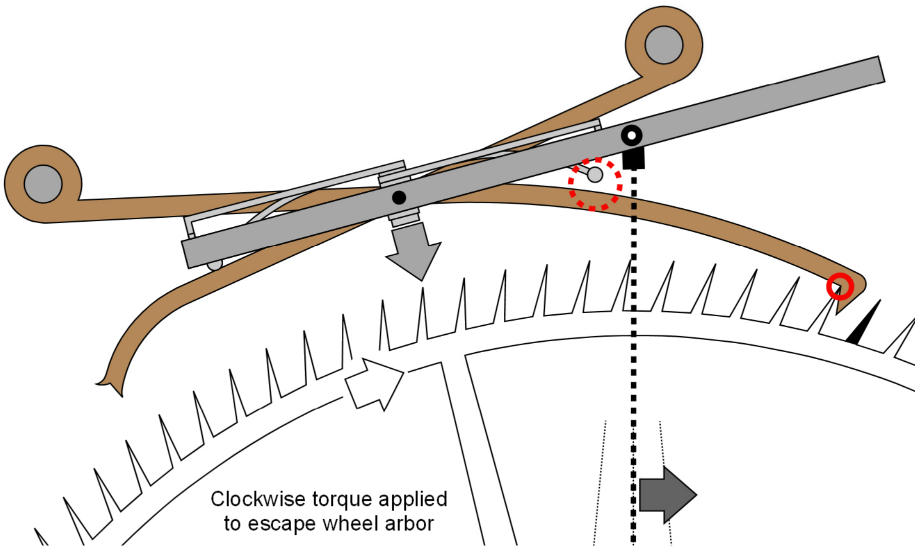


Figure 6.29 - Exit pallet impulsing. Exit pallet arm detached from exit composer.

Figures 6.30 (next) and 6.31 (next but one) occur virtually simultaneously, although, for clarity, they are illustrated and described separately.

Figure 6.30 (next) - As the pendulum passes the right hand amplitude marker, the entry pallet locking corner contacts an escape wheel tooth tip. There may be an audible 'click' upon contact. The escape wheel is briefly halted and the entry pallet locking corner is captured.

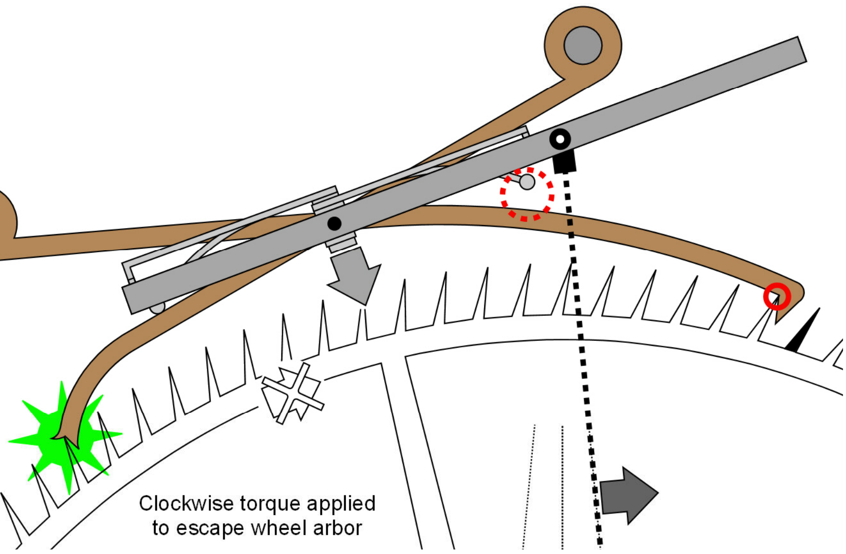


Figure 6.30 - Entry pallet capture. End of exit pallet impulse. Escape wheel halted.

Figure 6.31 (next) - Escape wheel impulse to the entry pallet applies clockwise torque about the escapement frame arbor. Diminishing pendulum momentum opposes that torque, during overswing to the right. The escape wheel is recoiled by the entry pallet, releasing the exit pallet. The exit pallet arm rotates anticlockwise about its pivot until arrested by the exit composer. There may be an audible 'click' as the exit pallet arm and composer meet. There may be further clicks if there is bounce. The entry pallet arm and composer rotate clockwise about the pallets pivot (illustrated entry composer changes from grey to green).

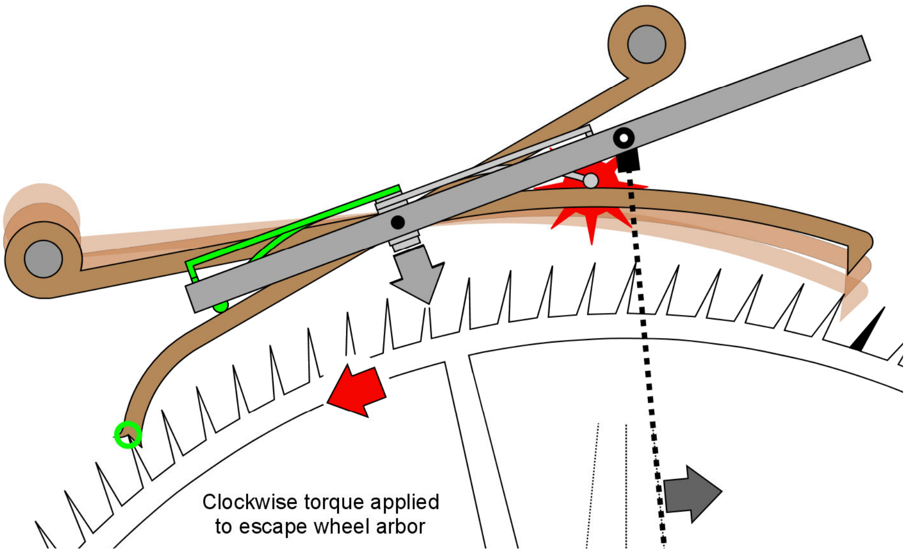


Figure 6.31 - Virtually coincident with Figure 6.30.
Escape wheel recoil and overswing begins. Exit pallet released. Entry composer begins to lift.

Figure 6.32 (next) - Pendulum momentum opposes entry pallet impulse. The pendulum and escape wheel will eventually stop, momentarily, at the limit of overswing to the right. The escape wheel is recoiled. The entry composer has been rotated clockwise about the pallets pivot, lifting its nose away from the escapement frame, as emphasised in the magnified view.

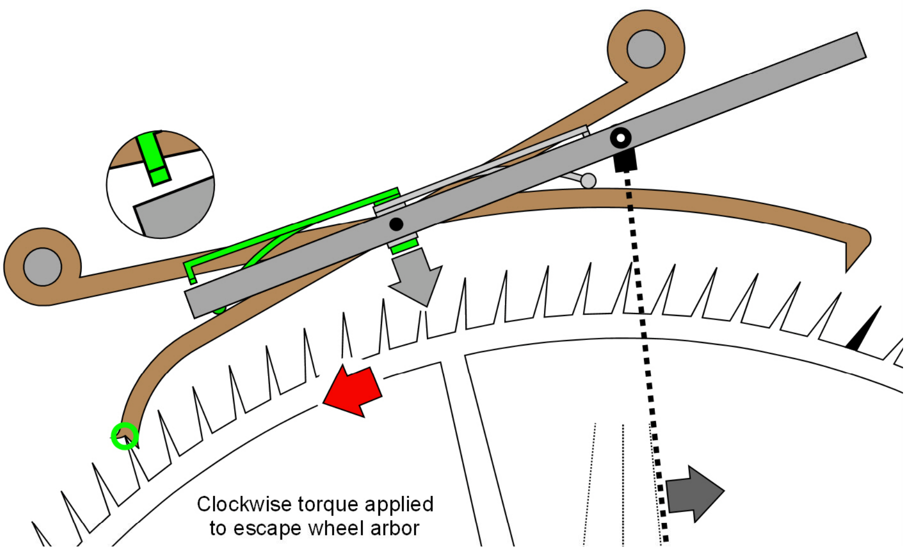


Figure 6.32 - Pendulum overswing. Escape wheel recoil continues. Entry pallet still captured.
Entry composer visibly clear of escapement frame.

CYCLE OF OPERATION
FOUR MINUTE ESCAPE WHEEL

The sequence of Figures 6.21 to 6.32 spans one complete cycle of operation, during which the dark escape wheel tooth has advanced clockwise by one full tooth space.

In normal operation, one complete cycle of operation would require two swings of the pendulum, i.e. from one extremity to the other and back again, which would occupy two seconds, when executed by a seconds beating pendulum. There are 120 teeth in the escape wheel, which will, therefore, require 2 x 120 = 240 seconds, or four minutes, to complete one full rotation. This may be conveniently referred to as a "four minute escape wheel'.

CYCLE OF OPERATION
NO FRICTION, LUBRICATION OR WEAR

If we now review all of the described stages involved in normal operation, we arrive at the startling conclusion that sliding friction is at all times completely absent. There is negligible sliding contact at the outer surface of the pallets pivot, as the pallet arms and composers rotate through small angles, during the capture and release phases. Practical experience, which is ultimately the real test, confirms that the pivot pin and pallet arms should be capable of operating continuously for many centuries without lubrication or noticeable wear, given suitable choices of materials and correct manufacture. Here, then, is an escapement requiring absolutely no maintenance whatsoever and suffering virtually no wear over centuries of operation. It is sobering to consider that a century of continuous service would involve just over 3155 million beats of the pendulum and that we could expect reliable, continuous operation for at least three of those centuries without lubrication, maintenance or any degradation in function.

PERFORMANCE STIPULATIONS

In 1775, the year before his death, Harrison recorded much of his life's work in a remarkable and immensely significant manuscript entitled :

'A DESCRIPTION CONCERNING SUCH MECHANISM AS WILL AFFORD A NICE, OR TRUE MENSURATION OF TIME; TOGETHER WITH SOME ACCOUNT OF THE ATTEMPTS FOR THE DISCOVERY OF THE LONGITUDE BY THE MOON; AS ALSO AN ACCOUNT OF THE DISCOVERY OF THE SCALE OF MUSIC'.

That document will henceforth be referred as 'CSM' (from ‘CONCERNING SUCH MECHANISM’).

CSM, included detailed stipulations for the design of all future timepieces adhering to his principles. Included amongst those stipulations were precise requirements for the performance of his grasshopper escapement.

As with much of his work, Harrison gives scant proof, if any, of his conclusions, many of which were almost certainly derived from practical experience and experimentation. It should not be inferred from this that he was lacking mathematical ability, for that is certainly not the case, but it does serve to explain his tendency to state conclusions without supporting, written evidence. In fairness, Harrison's willingness to devote time and effort to the production of manuscripts in any form is to be commended and we should be grateful for what we have. Nevertheless, to those who understandably find Harrison's approach difficult to accept, it may be reassuring to learn that, in all of his work, he demonstrated complete devotion to establishing absolute truth. In some instances, most notably in his efforts to create a viable longitude timepiece, his ruthless honesty was, it could be argued, to his own severe disadvantage, most especially in financial terms. At this stage in this publication, therefore, we shall accept Harrison's CSM statements, until evidence or doubts to the contrary arise. As will be appreciated, a remarkable aspect of the grasshopper escapement is that it may be adapted to satisfy virtually any realistic performance requirements, if desired. Our adherence to Harrison's CSM stipulations is, therefore, by no means irreversible.

It should be bourne in mind that many other CSM stipulations, encompassing all aspects of the design, setup and operation of an entire Harrison regulator, must be met in order to create a timepiece capable of the intended performance. In short, any grasshopper escapement, however perfect it may be, would be less effective and quite possibly inappropriate, in isolation from Harrison's other devices.

A relevant summary of Harrison's CSM stipulations is listed next. The numerical ordering is peculiar to this publication, for ease of reference, and should, most emphatically, neither be attributed to Harrison nor assumed to indicate any particular order of importance. Other stipulations, requirements and statements are presented in CSM; however, since they have no *currently* apparent influence upon the design or optimisation of the escapement, they will not be listed or described. We must, however, be receptive to, but cautious of, future interpretations of CSM that may conflict with this view; CSM can be intricate and may well hold many undiscovered secrets, but that intricacy and potential renders it susceptible to erroneous, misleading and/or deliberately false interpretations.

The reader may, quite understandably, be somewhat daunted and, perhaps confused by two of the stipulations listed below, numbered 2 and 3. A subsequent section, entitled TORQUE ARMS, RATIOS AND CIRCLES, will carefully explain their meaning at an arguably more appropriate point.

- **Stipulation 1 - There must be no sliding friction and no requirement for lubrication.**

This achievement has already been demonstrated and explained in the previous section.

- **Stipulation 2 - The mean torque arm must be one hundredth of the equivalent pendulum length.**

The term 'mean torque arm' requires clarification, which will be presented later, at a more appropriate point. In simple terms, Harrison is restricting the influence of the escapement, in order that the natural, free swing of the pendulum is not interfered with to an excessive degree.

The equivalent pendulum length is defined as the length of an idealised pendulum having point suspension, no rod mass and all pendulum mass concentrated at the centre of a circular bob. The period of such a pendulum must match that of the pendulum fitted to the timepiece in question. The equivalent length is the distance from the suspension point to the centre of the bob. A later section will include further illustration.

- **Stipulation 3 - The mean torque at the start of impulse must be two thirds* (2/3 or 0.6666 recurring) of the mean torque at the end of impulse, over one complete cycle of operation.**

The term 'mean torque' will be explained when appropriate. In simple terms, Harrison is defining the way in which escapement impulse should vary during each swing of the pendulum.

**In fact, CSM is annoyingly (and unnecessarily) imprecise as to the exact ratio to be used, as will be discussed later.*

- **Stipulation 4 - The pendulum arc should be large, although fifteen degrees should not be exceeded.**

For the purposes of this publication, eleven degrees will be assumed to be the ideal. That choice is not irreversible, for the design methods presented herein will permit the specification of any chosen pendulum arc, if so desired. The grasshopper escapement has a rare capacity to work at extremely large amplitudes with absolutely no degradation in performance. Suspension cheeks are, however, essential (and an extremely useful ally).

- **Stipulation 5 - A 'long pendulum' must be incorporated.**

All of Harrison's pendulums were seconds beating (or, to be precise, at or close to seconds beating). Seconds beating pendulum lengths will be assumed herein.

- **Stipulation 6 - A four minute escape wheel shall be used.**

An understanding of this stipulation will require a study of the later OBSERVATIONS section.

GEOMETRICAL REPRESENTATION

Although the ingenuity and elegance of the grasshopper escapement should certainly be apparent, to some extent, we are now in a position to investigate an aspect of the mechanism that may certainly be described as truly beautiful: the geometry. The first objective is to create a representative geometrical model. Fortunately, Harrison's stipulations provide invaluable clues as to what elements a geometrical construction must at least incorporate: Stipulation 1 dictates that we must avoid any sliding contact between components, Stipulations 4 and 6 are easily incorporated and Stipulations 2 and 5 merely define the size of the geometry, not its form. Stipulation 3 relates to the way in which escapement impulse varies during each cycle, and is of considerable influence. Unfortunately, significant effort is demanded of precise incorporation, as will be appreciated shortly.

Before proceeding further, it must be understood that, although CSM Stipulation 3, as listed herein, specifies a mean torque ratio of 2 to 3, Harrison actually states, with an uncharacteristic lack of precision, that that the ratio must be 'about as two' to three. We may never know, for certain, what 'about as two' was intended to mean: should it be less than two, or greater? In view of that uncertainty, the only logical, sensible and safe course of action must be to adopt a mean torque ratio of *precisely* 2 to 3. Such a policy unavoidably incorporates a possible deviation from Harrison's intention. However, should any ratio other than 2 to 3 be specified, that guess (for it would certainly be nothing other than a guess) might depart even further from the ideal. Another interpretation of Harrison's vagueness is that the mean torque ratio is not critical. This is especially unlikely, given the influence of the torque ratio in reducing circular error and enabling circular, rather than cycloidal, pendulum suspension cheeks. Reassuringly, the devised design tools, presented later, permit the retrospective incorporation of any desired mean torque ratio, should firm evidence of Harrison's exact intentions, or future proof of a better alternative, emerge.

It is useful, at this point, to mention that Harrison produced a single sheet of drawings, referred to herein as 'MS 3972/3', displaying four separate, partial or complete single pivot grasshopper escapement geometries.

A feature of MS 3972/3, certainly intentional (if only as personal aides mémoires), is that, for each geometry, Harrison has drawn early, independent lines and arcs of considerable length, whereas later, dependent lines and arcs terminate where they intersect prior constructions. In addition, he includes three single digits. Those features, with a little thought, reveal the sequence of construction and Harrison's thought process. The four geometries will be labelled, for convenience of description, as geometries '1', '2', '3' and '4'. Geometry 4 clearly represents Harrison's final attempt on that sheet of drawings, although it is not necessarily an ideal solution.

Unfortunately, MS 3972/3 bears no written confirmation whatsoever of its contents, intended purpose or origins; the entire sheet of drawings is totally bereft of any text. On that basis, in combination with uncertain drawing inaccuracy, potentially incomplete iteration and an absence of clearly stated conclusions, it must not be assumed that any of the geometries of MS 3972/3 are precisely as Harrison intended of all grasshopper escapements. Faithful adherence to, or logical interpretation of, the stipulations within CSM, which is a detailed, signed manuscript, must claim absolute priority.

Only geometry 4 spans 17.5 teeth of a 120 tooth escape wheel. That escapement span and total number of escape wheel teeth corresponds to an optimum solution, for a similar pendulum amplitude, identified by precise mathematical analysis (presented later). The measured mean torque arm ratio is 0.6837 (to 4 decimal places, but subject to copy distortion and measurement errors), which is close enough to 0.6666 recurring (i.e. 2/3) to suggest that Harrison is attempting to incorporate CSM Stipulation 3, or a variation of that stipulation.

It would appear that the scale of geometry 4 may have been adjusted, possibly in an effort to comply with Stipulation 2. An agreement between original MS 3972/3 dimensions and calculated equivalents for a seconds pendulum would confirm that suggestion. There may have been an attempt to match the geometry to an existing movement (or vice versa), quite possibly Harrison's Final Regulator. Frustratingly, at the time of writing, an appalling lack of published or available dimensions from MS 3972/3 or Harrison's regulator prevents any useful confirmation or exploration of those possibilities. Nevertheless, the MATHEMATICAL MODEL OUTPUT section, presented shortly, enables an intelligent comparison of factored MS 3972/3 dimensions and corresponding, mathematically derived sizes for mean torque arm ratios of both 0.6837 and 0.6666 recurring (2/3).

Significantly, analysis and measurement of MS 3972/3 suggests the presence of valuable features: **right-angles**. Geometries 1, 2, 3, and 4 include pallet arms with lines of action, at the start of impulse, tangential to the escape wheel (i.e. at right-angles to the applicable escape wheel radial through the pallet nib locking corner). Furthermore, geometries 3 and 4 incorporate right-angles between the escapement frame arbor, pallets pivot at mid-travel and escape wheel axis. Copying process distortion of the available drawing is assumed to be responsible for very slight deviations from the perpendicular, in some cases. Sensible practices firmly support the incorporation of right-angles at all of the described locations and more, whilst there wouls appear to be no valid reasons why they should be otherwise.

Figure 6.33 (next) introduces the geometry, derived from CSM and MS 3972/3, in optimised form, with a semi-transparent overlay of the mechanical components at the start of entry pallet impulse. By intention, the mechanical form illustrated in Figures 6.01 to 6.32 matches the geometry. It is absolutely vital to appreciate that the geometry represents a combination of every single one of the situations depicted in Figures 6.21 to 6.32, with the exception of overswing. Although the mechanical overlay of Figure 6.33 depicts only one of those situations, it usefully demonstrates what the geometry represents. It is important to realise that the illustration is an enlarged version of the physical reality. The 120 tooth escape wheel of this particular example would, in reality, be just under 145 mm in diameter, when used in combination with a seconds beating pendulum.

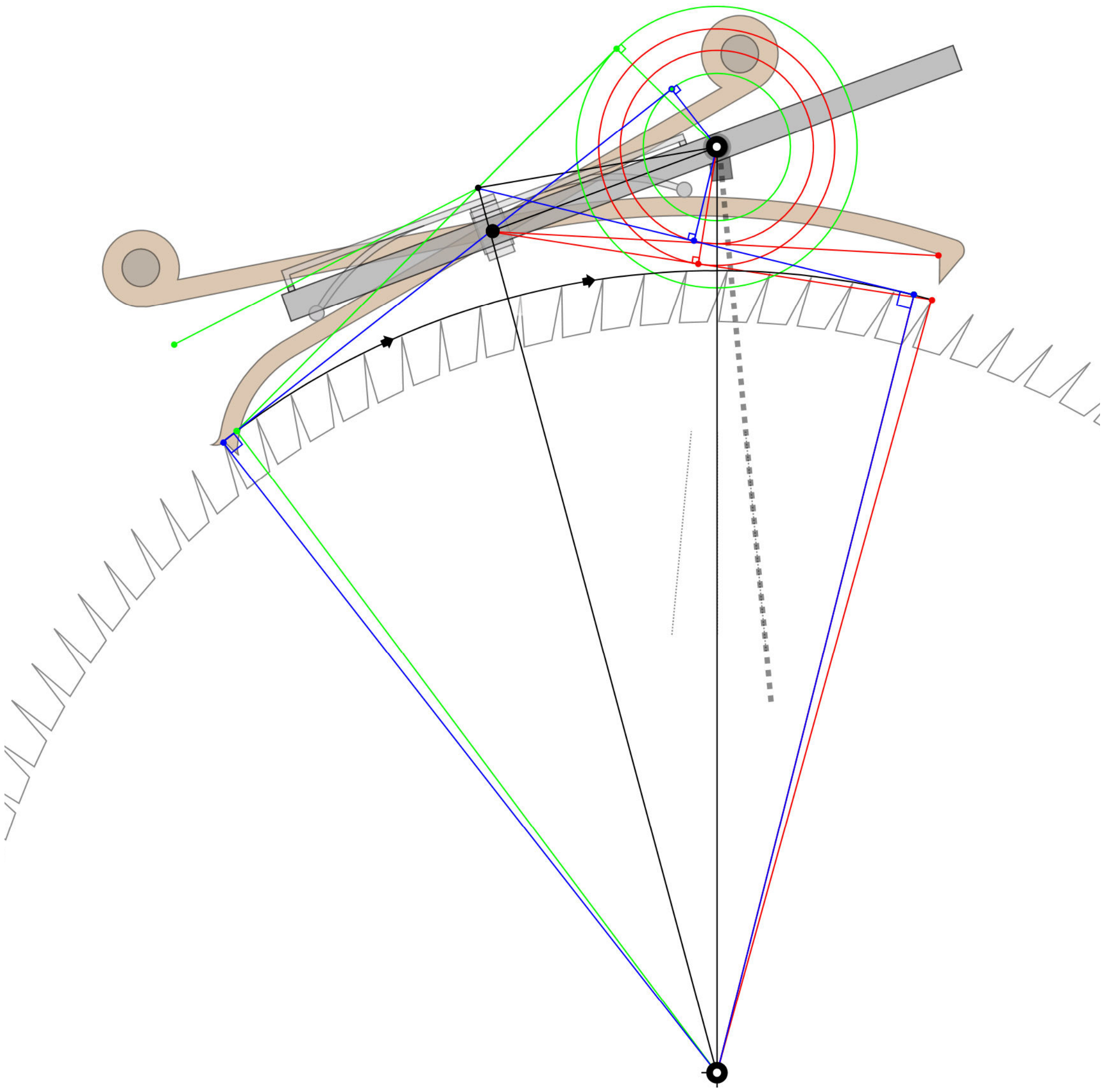


Figure 6.33 - Introductory single pivot grasshopper escapement geometry with a sample mechanical overlay.

Figure 6.34 (next) presents the geometry with the mechanical overlay removed. This figure will henceforth be used in preference to Figure 6.33, since it provides a clear, uncluttered view, suited to detailed analysis. The figure has been enlarged as much as the page width will permit, and annotated, in preparation for subsequent descriptions and explanations. Green lines are entry constructions, red are exit and blue is merely included (in this instance only) where confusion would otherwise result from the use of green or red. A guide is provided (on the next page), explaining what each point and angle represents.

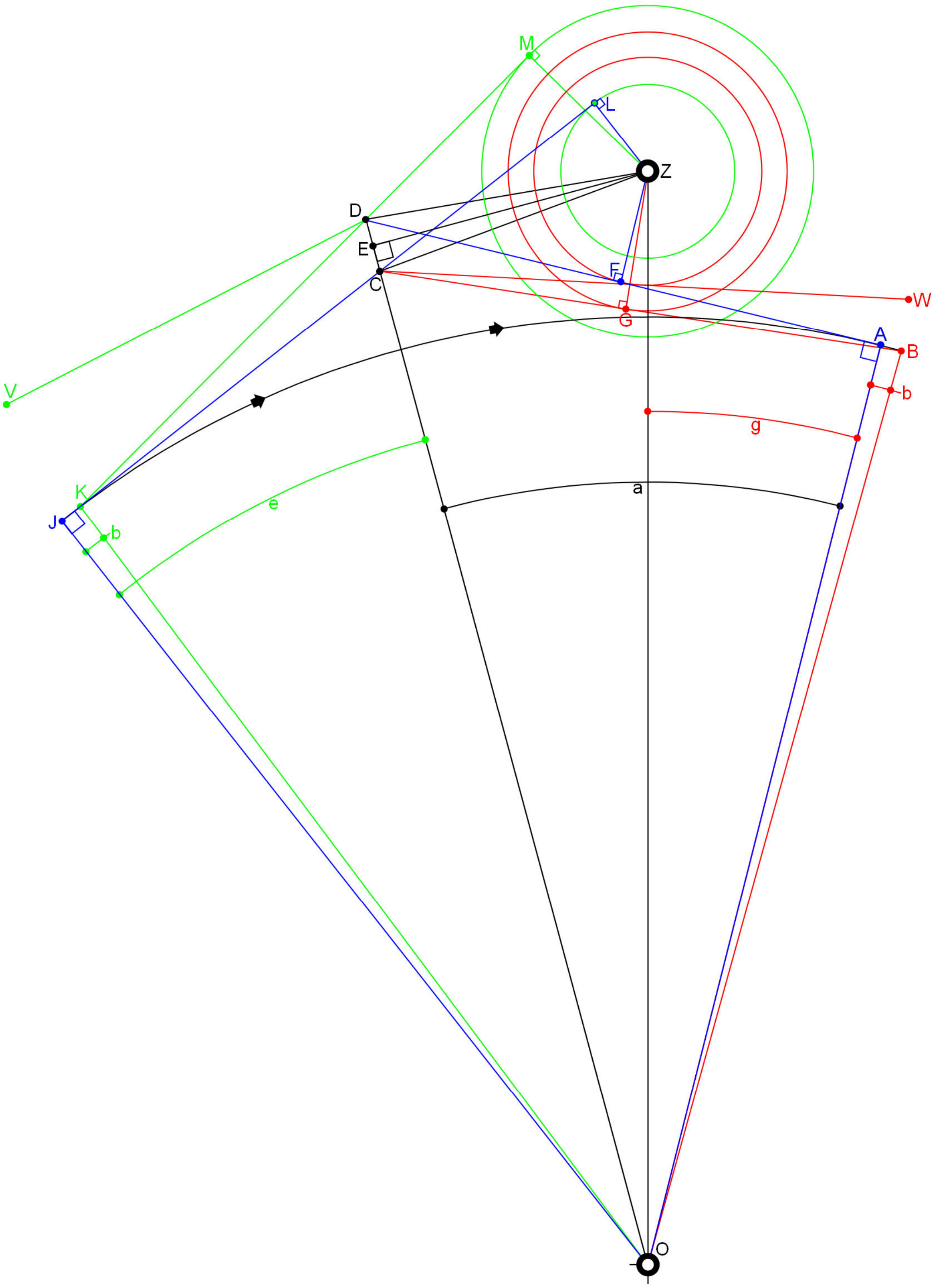


Figure 6.34 - Annotated single pivot grasshopper escapement geometry, to be used for the analysis.

GUIDE TO FIGURE 6.34
(in alphabetical order)

A - Exit pallet locking corner upon capture (start of impulse). AD is perpendicular to AO.

b - The angle subtended at O by half a tooth space.

B - Exit pallet locking corner at the start of release (end of impulse).

BW - Exit pallet travel ('lift') during release.

C - Pallets pivot. Travelling pivot (moves with the escapement frame).
The position shown corresponds to capture of the entry pallet and, therefore, release of the exit pallet.

Circular arc through , J, K, A and B - Locus of escape wheel tooth tips.
Escape wheel rotates clockwise about O.

D - Pallets pivot. Travelling pivot (moves with the escapement frame).
The position shown corresponds to capture of the exit pallet and, therefore, release of the entry pallet.

E - Bisects CD. EZ is perpendicular to OD.

a + e - The 'span' of the escapement, in degrees, subtended at O.
The use of two separate angles corresponds to the later mathematical analysis.
More commonly expressed in tooth spaces, as a whole number of tooth spaces plus half a tooth space.
Observe that angle 'a + e' is spanned by the circular arc between A and J, and that the circular arc between B and K spans the same angle (merely shifted clockwise about 'O' through angle 'b').

FZ - Exit pallet torque arm at the start of impulse (see later explanation, within the text).

g - Angle between separate escape wheel radials through A and Z

GZ - Exit pallet torque arm at the end of impulse (see later explanation, within the text).

J - Entry pallet locking corner upon capture (start of impulse). CJ is perpendicular to JO.

K - Entry pallet locking corner at the start of release (end of impulse).

KV - Entry pallet travel ('lift') during release.

LZ - Entry pallet torque arm at the start of impulse (see later explanation, within the text).

MZ - Entry pallet torque arm at the end of impulse (see later explanation, within the text).

O - Escape wheel arbor. Anchored pivot (attached to an immovable object).

V - Entry pallet locking corner at the end of release (resting upon entry composer).

W - Exit pallet locking corner at the end of release (resting upon exit composer).

Z - Escapement frame arbor. Anchored pivot (attached to an immovable object).

TORQUE ARMS, RATIOS AND CIRCLES

A complete understanding of Figure 6.34 requires clear explanations of the terms 'torque arm', 'torque ratio', 'torque circle' and associated terminology. This section presents those explanations.

- **ENTRY PALLET ARM.**

In Figure 6.34 the line CJ represents the most direct connection between the entry pallet locking corner at the start of impulse (first capture) and the corresponding position of the pallets pivot pin. Although the physical pallet arm will obviously differ in form from a simple straight line, its shape is irrelevant to the geometrical connection of those two points. Both end points are frictionless pivots (ignoring the negligible friction at the pallets pivot). Thus, the component, along CJ, of any force applied at C, will be transmitted along CJ to the other end point, J, where it will emerge, unaltered in magnitude by the journey and, most importantly, still in the same direction as line CJ. Any other components of the applied force will not be transmitted.

In order to determine the torque generated about the escapement frame arbor, Z, by the force along CJ, we must extend the line of the force along CJ until a perpendicular to that line passes through Z. To clarify, in Figure 6.34 the perpendicular in question is shown as the line LZ. The torque about Z will, therefore, be the the force along CJ multiplied by the distance LZ.

LZ is referred to as the **'torque arm'** of the entry pallet arm, CJ, at the start of impulse (first capture).

By the same reasoning, MZ is the torque arm of the entry pallet arm, DK, at the end of impulse (release).

The **'torque arm ratio'** of the entry pallet arm is defined as the ratio LZ/MZ.

For future reference, two circles may be constructed, as shown, centred upon Z and of radii LZ and MZ. These will be referred to as **'torque arm circles'**, for obvious reasons. Their purpose will become clear later.

- **EXIT PALLET ARM.**

In Figure 6.34, the line AD represents the most direct connection between the exit pallet locking corner at the start of impulse (first capture) and the corresponding position of the pallets pivot pin.

FZ is the torque arm of the exit pallet arm, AD, at the start of impulse (first capture).

GZ is the torque arm of the exit pallet arm, BC, at the end of impulse (release).

The torque arm ratio of the exit pallet arm is the ratio FZ/GZ.

Torque arm circles are constructed, centred at Z, of radii FZ and GZ.

The **'mean torque arm'** is the mean (average) of LZ, MZ, FZ and GZ. Stipulation 2 requires that the mean torque arm be 1/100th of the effective pendulum length. A seconds beating pendulum located in London, England, must be 994.156mm in effective length, which defines the associated mean torque arm as 9.94156mm. That mean torque arm may be achieved by enlarging or reducing the entire escapement and escape wheel until the resultant mean of LZ, MZ, FZ and GZ is 9.94156mm.

Stipulation 3 requires that the mean torque ratio be 2/3. If it is assumed that the force along each pallet arm is constant, we may equate the mean torque ratio to the mean torque *arm* ratio. For absolute precision, the effect of variations in the force along each pallet arm (as the pallet arm deviates from being tangential to the escape wheel at the start of impulse) may be incorporated at a later stage. The mean torque arm ratio is the mean (average) of LZ divided by MZ and FZ divided by GZ. Incorporating Stipulation 3 is, unfortunately, an extremely involved task, demanding a significant volume of detailed analysis and explanation. Two methods will be presented, in separate sections. The first method, graphical, is somewhat tedious, but will serve to identify the effects of altering significant parameters. The second method, mathematical, will significantly ease the design process and provide essential precision.

GRAPHICAL DESIGN PROCESS

A graphical design method was quite probably used by John Harrison. It must be conceded, however, that the only evidence of such is the single, solitary layout drawing, MS 3972/3, described previously. Nevertheless, it is extremely unlikely that he would have used a computational method, despite more than adequate skills in such disciplines.

Hand-drawn graphical methods, however carefully, skilfully and correctly performed, will inevitably introduce inaccuracies in drawing and in measurement. In addition, as will soon be appreciated, the graphical methods presented herein can be exceptionally tedious, in that a considerable degree of trial-and-error is required, albeit minimised by adopting an intelligent approach. Those who are familiar with Computer Aided Design (CAD) will benefit from the advantages of enhanced precision, although the author has experienced some deficiencies and inconsistencies when lines or arcs intersect at acute angles.

Apart from the undoubtedly beautiful and fascinating nature of the subject, the presentation of a graphical design method serves to clearly demonstrate how designer chosen demands may be met and which designer chosen parameters affect which final outcomes. Any sensible designer attempting to create a grasshopper escapement, by whatever method, should ensure that they are very familiar with such causes and effects if they wish to avoid an unnecessarily lengthy and frustrating process.

It is strongly recommended that the entire design process, SUMMARY and CONCLUSIONS be read and understood before attempting any independent graphical design.

For the example used to present the graphical design method, we shall assume that our designer is John Harrison and that, as might be expected, he is specifying parameters in strict accordance with his CSM stipulations, listed earlier. Such an approach will simplify the concurrent tasks of explaining the graphical method and describing Harrison's stipulations in detail. Nevertheless, guidance will also be given in the methods involved in incorporating any desired stipulations.

Figures 6.35 to 6.41 illustrate the graphical design process in chronological order. Early figures will be presented to a scale best suited to the layout of the explanation. In order to demonstrate and illustrate the incorporation of Stipulation 2 (i.e. the mean torque arm must be 1/100th of the equivalent pendulum length), later figures will be as close as possible to a scale of one to one (full size). All will become clear as the process unfolds.

The illustrations will use green to represent fresh escapement entry constructions and red to represent fresh exit constructions. When fresh constructions are neither entry nor exit orientated, they will be shown in blue.

As each new figure in the sequence is presented, previously green or red coloured elements will change to black, in order to indicate that they are no longer fresh constructions, but are carried over from a previous illustration.

When creating new escapement geometries, designers are advised to repeat the sequence presented herein to as large a scale as available drawing equipment will permit, in an effort to minimise drawing and measurement inaccuracies. For the same reasons, it is suggested that, unless a large and accurate protractor is available, angles should be produced by resolving them (using the trigonometric function 'tan') into the two shortest sides of the largest practicable right-angled triangles and laying out those sides vertically and horizontally, as applicable.

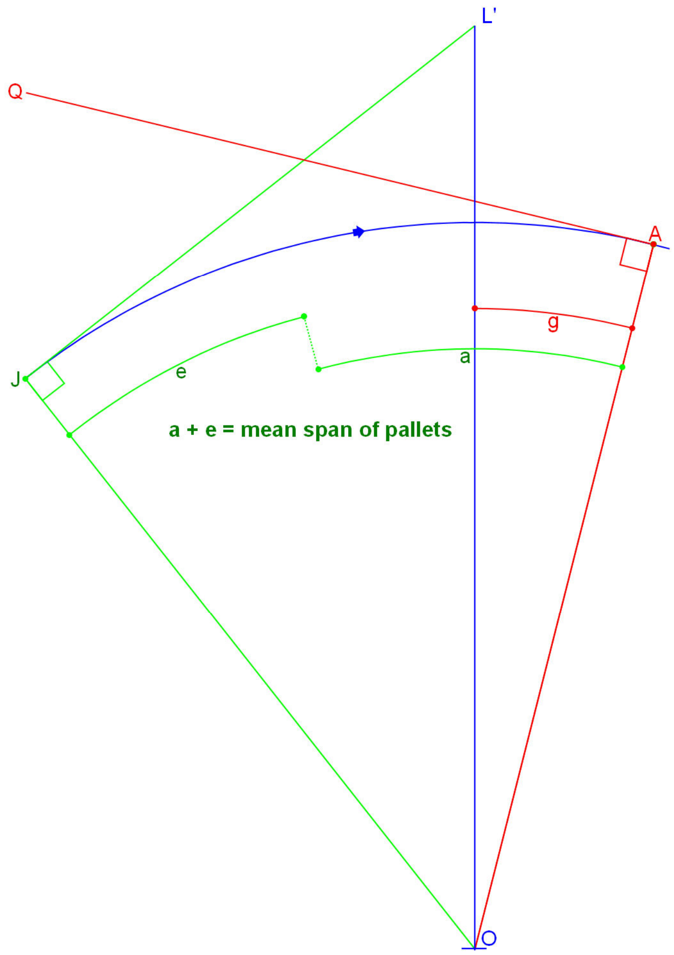


Figure 6.35 - Graphical Construction - Step 1

STEP ONE

Mark a point, O, at the bottom of the page, slightly right of centre. Draw a vertical line through point O, as long as the page will allow.

Draw an arc, centred at O, representing the escape wheel tooth tips pitch circle. The radius may be arbitrary, but should broadly be to the illustrated proportions.

Choose (estimate) angle g, clockwise from the vertical through OL'. Draw OA. Early estimates of angle 'g' will probably require adjustment later. The proportions illustrated offer sensible (albeit, at this early stage, unavoidably imprecise) guidance.

Choose the mean number of escape wheel teeth to be spanned by the pallets (whole number of tooth spaces, plus half a tooth space). Convert to an angle, in degrees. That angle is 'a + e'.

Draw OJ, anticlockwise removed from OA by angle 'a + e'.

Draw JL' perpendicular to OJ. This is the direction of the applied force, in compression, transmitted by the entry pallet arm at the start of impulse.

Draw AQ perpendicular to OA. This is the direction of the applied force, in tension, transmitted by the exit pallet arm at the start of impulse.

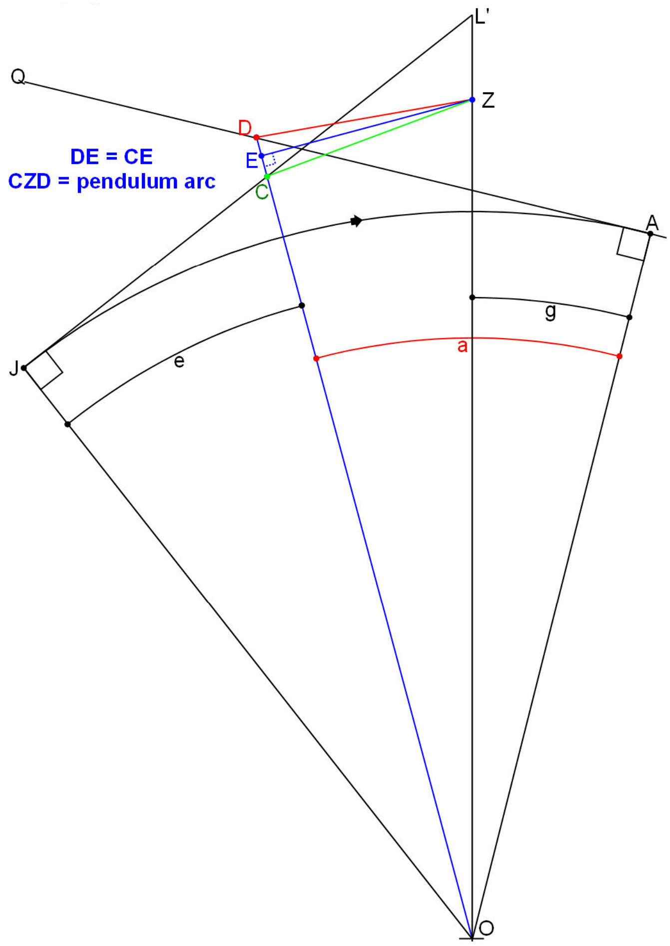


Figure 6.36 - Graphical Construction - Step 2

STEP TWO

Estimate the position of and draw line OD such that it creates separate points C and D, anticlockwise removed from the intersection of AQ and JL'. Observe that OD forms angle 'a' with OA.

C is an estimate of the entry pallet pivot position at the start of impulse and D is an estimate of the exit pallet pivot position at the start of impulse.

Bisect CD, creating point E. Draw EZ, perpendicular to OD. Z is an estimate of the position of the escapement frame arbor axis.

Angle CZD is the resultant pendulum arc (ignoring overswing). Although the pendulum arc may well be altered during a later step, there is little point in pursuing a position of OD that results in a pendulum arc far from the desired figure. The position of OD (i.e. the value of angle 'a') should, therefore, be altered and STEP TWO repeated from the beginning, until a satisfactory pendulum arc is broadly achieved.

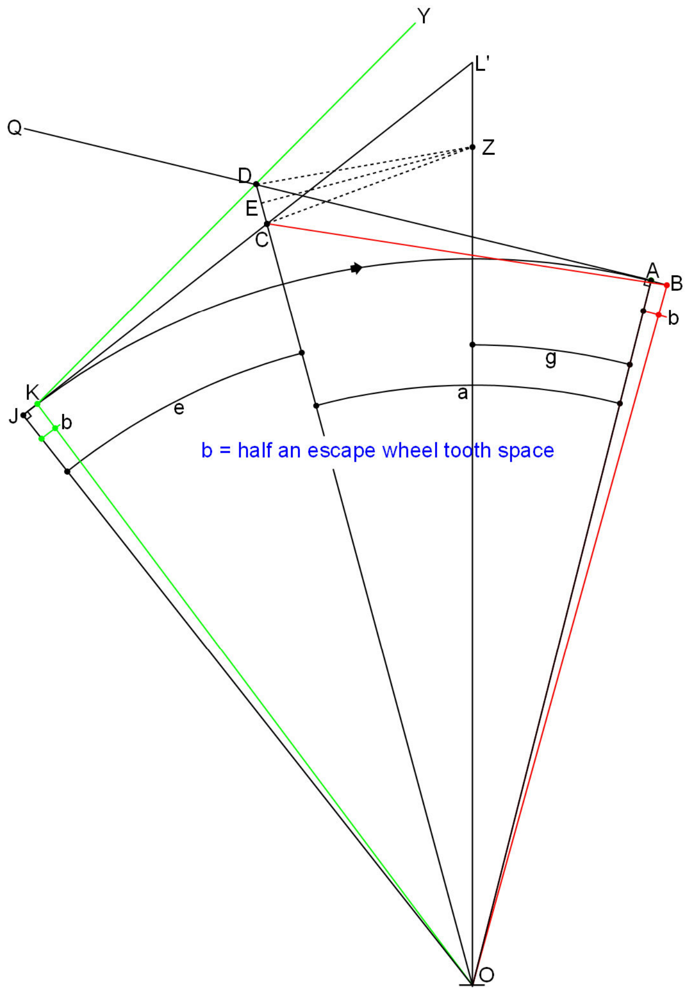


Figure 6.37 - Graphical Construction - Step 3

STEP THREE

Draw OK half a tooth space (angle b) clockwise from J. K is the entry pallet locking corner at the end of impulse.

Draw OB half a tooth space (angle b) clockwise from A. B is the exit pallet locking corner at the end of impulse.

Draw line from K through D, extending to Y. Line KY is the direction of the applied force, in compression, produced by the entry pallet at the end of impulse.

Draw line from B to C. There is no value in extending this line beyond C. Line BC is the direction of the applied force, in tension, produced by the exit pallet at the end of impulse.

Determine whether CJ = DK and AD = BC (distances from the pallet locking corner to the pallet pivot for each pallet). If either or both equalities are not satisfied, which is likely during early attempts, then the position of OD is unsuitable and requires alteration (which is equivalent to altering angle 'a' without altering a + e). Continue making adjustments until CJ = DK and AD = BC.

Should the pendulum arc then be unacceptable, the mean number of teeth spanned (a + e) must be altered and the entire graphical design process repeated from the beginning of STEP 1. It will speed the process of iteration if note is made of the values of significant parameters, how changes in one affects others and how other parameters, in turn, affect it. Trends may thus be identified, enabling intelligent revisions to be made. It may (most probably will) eventually be concluded that a solution can only be found if the pendulum arc is allowed to deviate from the desired value, at which point a decision as to the amount and sense (increase or decrease) must be made.

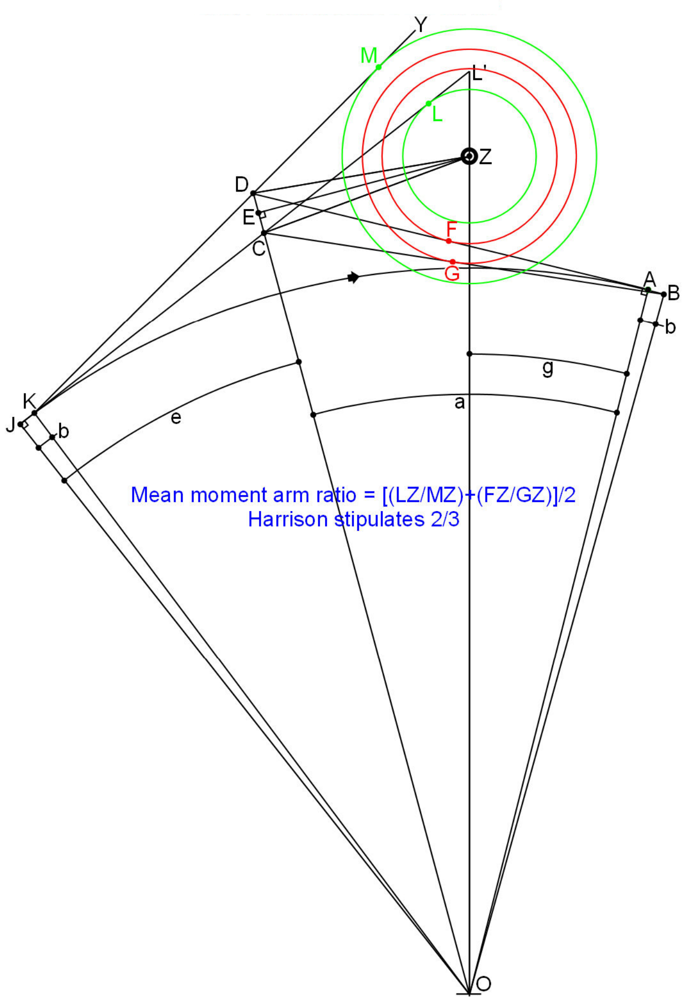


Figure 6.38 - Graphical Construction - Step 4

STEP FOUR

Draw four torque arm circles, centred at Z, tangential to (touching, at only one point) lines JL', KY, AD and BC .

Determine lengths LZ, MZ, FZ and GZ (measuring the diameter of each torque arm circle and dividing by two is a convenient method).

Calculate the torque arm ratios, LZ / MZ (entry pallet) and FZ / GZ (exit pallet).

Calculate the mean torque arm ratio, which is (LZ / MZ) added to (FZ / GZ), all then divided by two. The mean torque arm ratio must be as desired (Harrison stipulates 2/3). If this is not the case, angle 'g' must be altered and the entire graphical design process repeated from the beginning of STEP 1.

If it eventually becomes clear that no value of 'g' will provide a satisfactory solution, the total number of escape wheel teeth must be altered and the entire graphical design process repeated from the beginning of STEP 1.

NB - for absolute precision, account should be taken of the variation in impulse force along each pallet arm, as the line of action deviates from being tangential to the escape wheel at the start of impulse. The derived torque ratio of each side of the geometry should be factored (multiplied) by the ratio of the applicable force at the end of impulse to the force at the start of impulse. SUGGESTED METHOD - Measure angles DKO and CBO. Draw two right angled triangles to as large an arbitrary scale as possible, one containing angle DKO-90, the other containing angle CBO-90. The length of the longest side (hypotenuse) represents the force at the start of impulse 'Fs'. The length of the slightly shorter side represents the force at the end of impulse 'Fe'. Start again from STEP ONE, using torque ratios *reduced* by multiplying the previous torque ratio by the applicable ratio of Fe to Fs (Fe/Fs being less than 1).

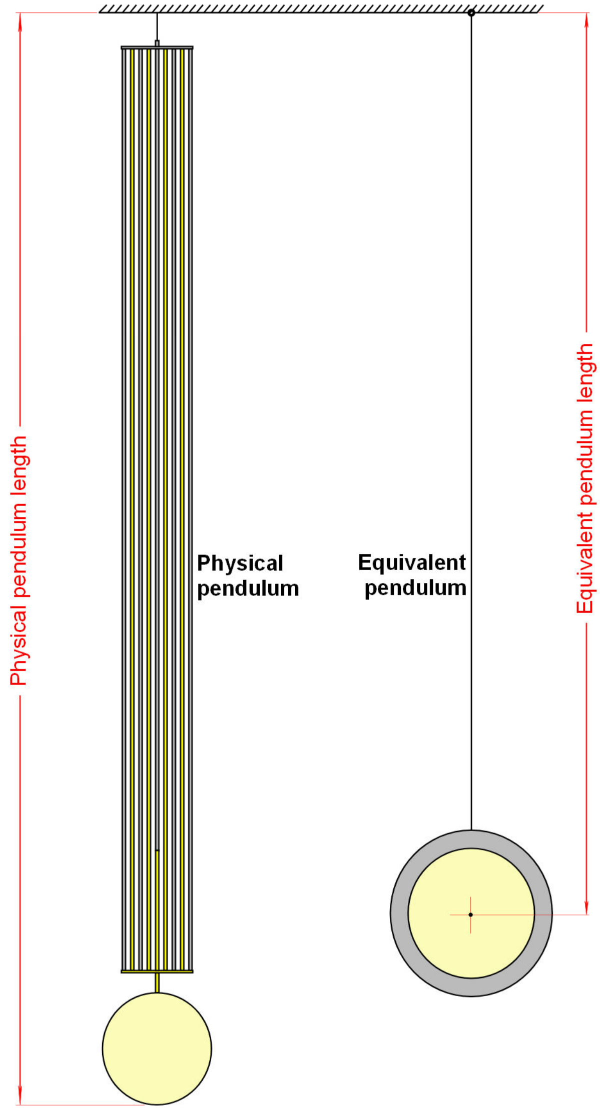


Figure 6.39 - Graphical Construction - Step 5

STEP FIVE

Determine the equivalent (idealised) pendulum length for the intended location of the timepiece.

The equivalent pendulum length is not the overall, physical length of the pendulum, but is a shorter, theoretical length. It is the distance from the point of suspension to the centre of gravity of an imaginary bob at which the entire mass of the pendulum is concentrated, as shown above, right hand illustration.

The equivalent pendulum length is a function of the local acceleration due to the Earth's gravity, which may be found listed in applicable publications. For example, in London, England, we may assume that a seconds beating pendulum will have an equivalent pendulum length of 994.156mm (39.14 inches).

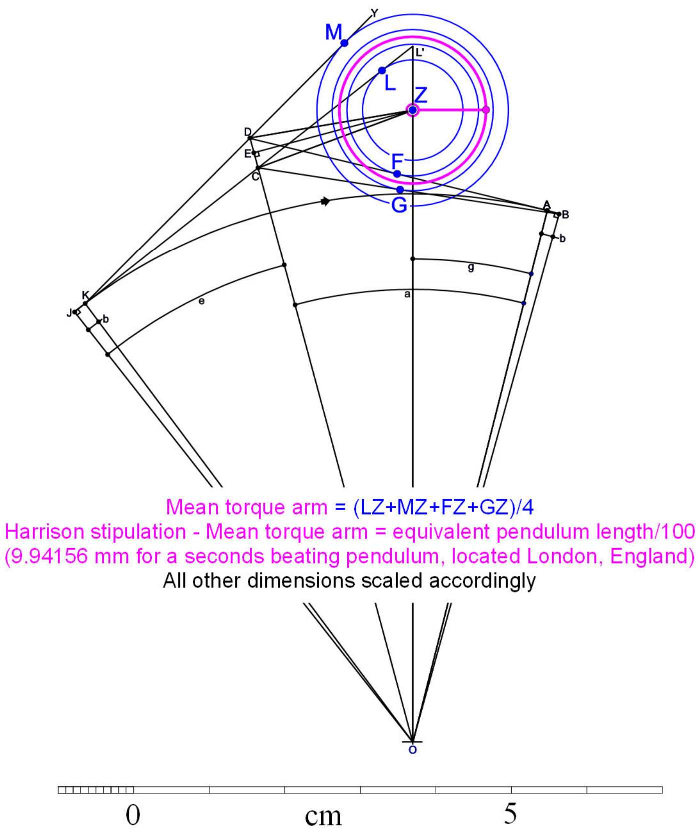


Figure 6.40 - Graphical Construction - Step 6

STEP SIX

It is assumed that a compliant geometry has been determined in the previous five steps.

(1) - Calculate the mean torque arm of the construction produced at the successful completion of STEP FOUR, by adding LZ, MZ, FZ and GZ (from STEP FOUR) and dividing the result by four.

(2) - According to Harrison Stipulation 2, the mean torque arm of the installed escapement must equal one hundredth of the equivalent, (idealised) pendulum length. For London, England, the required mean torque arm of the escapement should, therefore, be 994.156 divided by 100, which is 9.94156mm (0.3914 inches).

If that is not the case (as is likely), it will be necessary to multiply every linear dimension in the graphical construction by the required mean torque arm (i.e. 9.94156mm), divided by the existing mean torque arm (calculated in (1), above). Angles must not be altered.

It is recommended that the geometry be redrawn to the dimensions determined in (2) above, in order to produce a geometry of the same size as that of the installed escapement. Figure 6.40 is just such a drawing.

Designers may obviously substitute any desired ratio of equivalent pendulum length to mean torque arm.

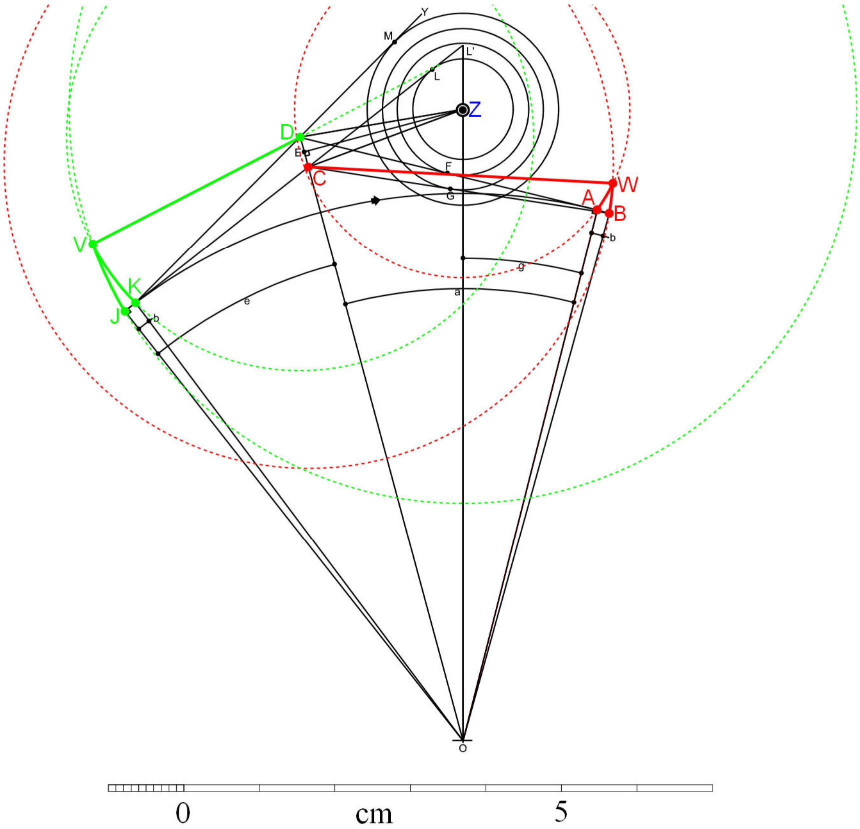


Figure 6.41 - Graphical Construction - Step 7

STEP SEVEN

Draw an arc of a circle, centred at Z, passing through J. Draw another arc of a circle, centred at D, passing through K. The point of intersection, V, of the two arcs, near K, represents the position of the entry pallet locking corner after release, assuming instantaneous resting of the entry pallet arm upon the entry composer. KV is, therefore, the entry pallet locking corner movement, or 'lift', upon release.

Draw an arc of a circle, centred at Z, passing through A. Draw another arc of a circle, centred at C, passing through B. The point of intersection, W, of the two arcs near B represents the position of the exit pallet locking corner after release, assuming instantaneous resting of the exit pallet arm upon the entry composer. BW is, therefore, the exit pallet locking corner movement, or 'lift', upon release.

Observe that lines DV, extended from D (broken line) and CW are tangential to the torque arm circles corresponding to the start of impulse and will continue as tangents until the next capture event (for the applicable pallet arm) takes place.

The purpose of the above constructions is to check that pallet nib lift would enable the production of a sensible escapement. Acceptable lift is, amongst other things, related to the anticipated quality of construction and the intended operating environment. Exit pallet locking corner lift will also dictate the maximum exit pallet nib length, which is a vital parameter when striving to reduce the possibility of escapement trip and escape wheel runaway.

Escape wheel tooth profile will be dictated by the exit pallet nib length. Inadequate escape wheel tooth strength and/or poor resistance to impact damage may demand a reduction in exit pallet nib length at the expense of reduced trip protection, or a reduction in escape wheel tooth count

GRAPHICAL DESIGN PROCESS
SUMMARY

(1) - For each individual pallet arm, the value of 'a' must be adjusted in order that the pallet arm lengths, from locking corner to pivot, will match at the start and end of impulse.

(2) - The value of 'a + e' may be used to adjust the pendulum arc.

(3) - The value of 'g' may be used to adjust the mean torque ratio.

(4) - For absolute precision, the variation in force along each pallet arm should be accounted for.

(5) - Should a solution prove to be impossible, the total number of escape wheel teeth must be adjusted.

(6) - The entire escape wheel and escapement geometry must be sized, in order that the mean torque arm is one hundredth of the equivalent pendulum length.

GRAPHICAL DESIGN PROCESS
CONCLUSIONS

It will now be appreciated that the graphical design method has the potential to be somewhat tedious. Although the process is certainly not 'hit-and-miss', provided that intelligent trend recognition is employed, it is obviously based upon an iterative process that can be extremely time consuming. Whilst being an inconvenience rather than an insurmountable obstacle, when such difficulties are combined with the disadvantages of drawing and measurement inaccuracies, the conclusion must be that a easier and more precise design process is required. Fortunately, such a process has been devised, as will be explained in the next sections.

MATHEMATICAL DESIGN PROCESS

For those who are averse to mathematics, all of the MATHEMATICAL DESIGN PROCESS sections may be completely ignored, with the notable exception of MATHEMATICAL MODEL OUTPUT, which presents a complete set of optimised geometries, enabling the relatively easy creation of any one of a useful range of single pivot grasshopper escapements. However, there is considerable pleasure to be gained and much beauty to be discovered within the mathematical design process and it would be a crime to forego those experiences for want of a little thought and effort.

For readers who are mathematically disinclined, but who are sufficiently computer literate, all that is required in order to apply the mathematical design process described herein is a basic understanding of computer spreadsheets, most especially the incorporation of formulae. The mathematical analysis is, by intention, presented in a manner particularly well suited to efficient conversion to spreadsheet format. Regrettably, dedicated spreadsheet instruction is beyond the scope and size of this publication and almost certainly differs, to varying degrees, between available software. For the same reason, a presentation herein of sample spreadsheets would serve no universal purpose. Furthermore, the compilation of working spreadsheets is quite straightforward, being nothing more than tedious at the input stage.

MATHEMATICAL DESIGN PROCESS
OBJECTIVES AND BASIS OF ANALYSIS

The first objective was to devise a straightforward method of creating optimised geometries of the single pivot grasshopper escapement.

A second objective was to achieve absolute precision. A mathematical approach, in combination with the precision of digital computation, promises absolute accuracy.

A third objective was to enable and encourage further research. It was, therefore, considered essential that a *universal* design tool should be devised. Such a tool would be capable of compliance with any set of constraints, rather than be limited to Harrison's CSM stipulations.

In preparation for the creation of the mathematical model, it is essential to identify those parameters that uniquely define the escapement geometry. In addition, the defining parameters require separation into those that should be chosen by the designer and those that must be mathematically generated.

Designer chosen parameters and values are identified as:

- Desired pendulum arc.
- Initial escape wheel tooth tip radius.
- Total number of escape wheel teeth.
- Mean number of escape wheel teeth spanned.
- Angles 'a' and 'g' (defined shortly)
- Equivalent pendulum length.
- Mean torque arm
- Mean torque ratio

Mathematically generated parameters are identified as:

- Final escape wheel tooth tip radius.
- Escape arbor to escapement frame arbor separation.
- Escapement frame arbor to pallets pivot separation.
- Entry pallet locking corner to pivot separation.
- Exit pallet locking corner to pivot separation.
- Entry and exit pallet locking corner lifts upon release.

The mathematical model is based, amongst other things, upon a realisation that the escapement layout may be separated into two geometries, unified at the escape wheel arbor, the escapement frame arbor and only two other points. To explain, **Figure 6.42 (next)** shows the single pivot grasshopper escapement geometry split along radial OD into two separate entities. For publications in colour, green will continue to indicate elements for the entry side of the escapement, red will indicate those for the exit side and blue will signify commonality. The figure to the left of Figure 6.42 is the geometry for the entry side and to the figure to the right is that for the exit side. The illustration serves to demonstrate that, in addition to the sharing of two common arbors, a defining connection between the two sides only exists at points C and D (i.e. the single pallets pivot at its extreme positions, ignoring overswing). The sharing of common arbors could apply to any static combination of entities, compatible or not, whereas the sharing of points C and D demands geometrical compatibility throughout the entire dynamic range of operation. The mathematical model incorporates that requirement by using entirely separate processes to determine the distance between C and D for both the entry and exit geometries. Other requirements aside, the entry and exit geometries will match when the distance CD calculated for the entry geometry equals the distance CD calculated for the exit geometry.

The two sides of the escapement may be further interrelated by the stipulation that the mean torque ratio must be as specified by the designer (e.g. Harrison specifies 2/3). For a chosen pendulum arc, that stipulation constrains the assembled geometry to a single solution.

The mathematical model is reliant upon an assumption the the line of force along each pallet arm at the start of impulse is tangential to the escape wheel at the location of the engaged pallet locking corner. As described in an earlier section, MS 3972/3 consistently indicates that such an assumption is valid.

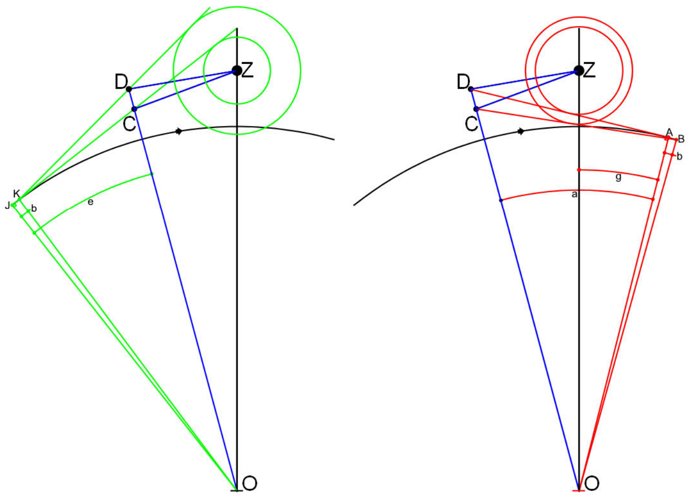


Figure 6.42 - Entry and exit geometries separated. Useful unification occurs at points C, D, O and Z.

MATHEMATICAL DESIGN PROCESS
MATHEMATICAL MODEL

Figure 6.43 (next) and the associated guide define the points and angles used throughout the mathematical modelling process. The geometry is precisely the same as that used to illustrate the graphical analysis, except for the addition of essential lines and annotated points of intersection.

Calculations, listed in a logical sequence, will be presented such that the purpose and derivation of every equation is absolutely clear. To that end, in addition to the normal progression of calculations from top to bottom of the page, their development will also flow from left to right, as required. Thus, an essential parameter will be identified and its most obvious method of calculation listed in what might be termed a 'primary' equation to the extreme left of the page. The subsequent determination of any unknown parameters within that primary equation will generate a requirement for one or more 'secondary' equation(s) , which will be listed below and immediately to the right of the primary equation. Unknown parameters within the secondary equation(s) will require one or more 'tertiary' equations, listed below and immediately to the right of the secondary equation. Equations with more than one unknown parameter will require more than one set of subsidiary equations. That process continues until all relevant parameters have been determined.

A sound understanding of the above method of presentation is essential. The following universal example will serve to emphasise the layout:-

PRIMARY EQUATION
　　　　SECONDARY EQUATION
　　　　　　　　TERTIARY EQUATION
　　　　SECONDARY EQUATION
　　　　　　　　TERTIARY EQUATION
　　　　　　　　　　　　ETC........(AS REQUIRED)

It is essential to note that, once a parameter has been determined, its derivation will not be repeated should it arise in subsequent calculations. Thus, if a parameter is apparently bereft of a deriving equation, that should be regarded as a cue to review preceding calculations in search of the derivation.

An obvious feature of the mathematical modelling technique is that, beyond the more obvious limits of physical possibility, impracticable designs can be generated, without necessarily being apparent. Be warned that neatly printed, computer generated figures do not guarantee correct information. In an effort to avoid damaging errors, the numerical results of escapement calculations should be physically drawn, to as large a scale as available drawing equipment will permit. That will confirm, within the limits of drawing accuracy, that the escapement design is practicable and will also permit confirmation that the required stipulations have been met.

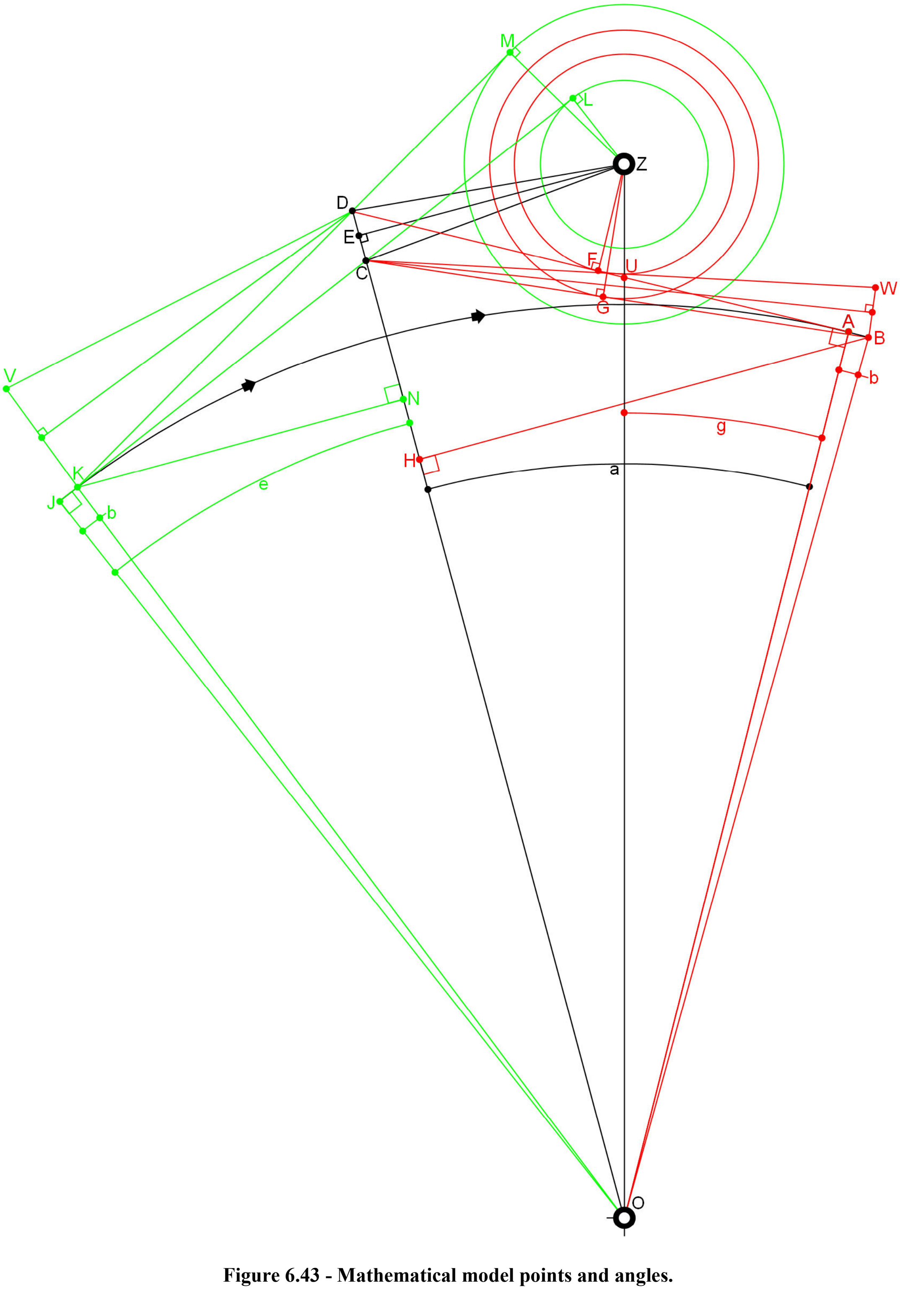


Figure 6.43 - Mathematical model points and angles.

GUIDE TO FIGURE 6.43 (parameters in alphabetical order)

In addition to all previously defined points (with some repetition):

'a' - designer chosen angle.

a+d - mean angle spanned by the pallet nib locking corners (see 'n' below).

b - half the angle subtended at O by two adjacent escape wheel tooth tips.

C - position of the common pallet pivot at the end of the impulse to the exit pallet (start of the impulse to the entry pallet). Recoil irrelevant.

D - position of the common pallet pivot at the end of the impulse to the entry pallet (start of the impulse to the entry pallet). Recoil irrelevant.

g - designer chosen angle.

L* - equivalent pendulum length (idealised pendulum) **-** designer chosen. Harrison specifies a 'long pendulum'. A seconds beating pendulum was clearly intended for Harrison Final Regulator.

M - mean torque arm of the escapement over one complete cycle of operation. for escape wheel radius R

M* - mean torque arm of escapement over one complete cycle of operation, for R= R*. (Harrison stipulates M* = L*/100).

n - mean number of escape wheel tooth spaces spanned by the pallet nib locking corners (whole number, plus a half) **-** designer chosen. Angle a+d expresses the span in degrees.

N - total number of escape wheel teeth - designer chosen.

O - escape wheel axis.

p - pendulum arc, from one extremity of swing to the other. Designer chosen, nominally.

R - designer chosen escape wheel teeth tips pitch circle radius.

R* - escape wheel teeth tips pitch circle radius complying with chosen stipulations. (Harrison specifies M* = L*/100).

t - mean torque ratio (or, more correctly, mean torque ***arm*** ratio) of the escapement over one complete cycle of operation. Harrison specifies 2/3.

t1 - entry pallet torque ratio (or, more correctly, mean torque ***arm*** ratio) over one complete cycle of operation.

t2 - exit pallet torque ratio (or, more correctly, mean torque ***arm*** ratio) over one complete cycle of operation.

Z - escapement frame arbor axis.

MATHEMATICAL DESIGN PROCESS
MATHEMATICAL ANALYSIS

CALCULATION (1) - Matching CD(entry) and CD(exit) along OD

CD(entry) = DN+NO-CO
DN = DKsinDKN
DK = CJ
CJ = JOtan(e)
JO = R
R = designer chosen
e = 2bn-a
b = 180 / N
N = designer chosen
n = designer chosen
a = designer chosen
DKN = arccos(KN / DK)
KN = KOsin(e-b)
KO = R
NO = KOcos(e-b)
CO = JO / cos(e)
CD(exit) = DO-CH-HO
DO = AO / cos(a)
AO = R
CH = BCsinCBH
BC = AD
AD = AOtan(a)
CBH = arccos(BH / BC)
BH = BOsin(a+b)
BO = R
HO = BOcos(a+b)

CALCULATION (2) - Complying with the designer stipulation for mean torque ratio - t (Harrison t = 2/3)

t = (t1+t2) / 2
t1 = LZ / MZ
LZ = CZsinLCZ
CZ = sqrt(CE^2+EZ^2)
CE = CD / 2
EZ = EOtan(a-g)
EO = CO+(CD / 2)
g = designer chosen
LCZ = 180-DCL-OCZ
DCL = JCO
JCO = 90-e
OCZ = 180-ECZ
ECZ = 90-CZE
CZE = arctan(CE / EZ)
MZ = DZsinMDZ
DZ = CZ
MDZ = DZE+DKN
DZE = CZE
t2 = FZ / GZ
FZ = UZcosFZU
UZ = OZ-OU
OZ = EO / cos(a-g)
OU = AO / cos(g)
FZU = g
GZ = CZsinGCZ
GCZ = 180-BCO-ECZ
BCO = 90-CBH

CALCULATION (3) - Pendulum arc - p

p = DZE+CZE

CALCULATION (4) - Accounting for the variation in force along each pallet arm during impulse.

t = [t1(new)+t2(new)] / 2
t1(new) = t1(old)sinDKO
DKO = NKO + DKN
NKO = 90-KON
KON = e-b

t2(new) = t2(old)sinCBO
CBO = CBH + HBO
HBO = 90-BOH
BOH = a+b

CALCULATION (5) - Complying with the designer stipulation for mean torque arm - M*

M* / M = R* / R
rearrange as R* = RM*/M
M = (LZ+MZ+FZ+GZ) / 4
M* = designer chosen (Harrison stipulates M* = L* / 100)

CALCULATION (6) - Pallet nib locking corner lifts immediately following release

Entry pallet lift = KV
KV = 2DKsin(KDV / 2)
KDV = MDZ-LCZ
Exit pallet lift = BW
BW = 2BCsin(BCW / 2)
BCW = BCZ-ADZ
BCZ = GCZ
ADZ = FDZ
FDZ = 180-MDZ-KDN-ADO
KDN = 90-DKN
ADO = 90-a

MATHEMATICAL DESIGN PROCESS
SUGGESTED SEQUENCE OF CALCULATIONS

(A) - Choose values for all designer-chosen parameters (a, g, L*, M*, n, N, p (target), R, t (target)).

(B) - Adjust angle 'a' until CD(entry) = CD(exit). See Calculation (1).

(C) - Adjust angle 'g' until t is as chosen. See Calculation (2). Repeat the analysis from the beginning of (B).

(D) - Adjust the mean number of teeth spanned, 'n', until an acceptable pendulum arc, 'p', is achieved. See Calculation (3). Subsequent to each adjustment of 'n', the analysis must be repeated from the beginning of (B).

(E) - To account for variation in force along each pallet arm during impulse (if required), determine the revised torque arm ratio, t(new). See Calculation (4). Repeat the entire analysis from the beginning of (B).

(F) - Multiply all linear dimensions by the ratio R* / R (e.g. Harrison stipulates that M* = L* / 100). See Calculation (5).

(G) - Calculate pallet locking corner lifts following release, based upon escapement dimensions derived from (F). See Calculation (6). If unacceptable, alter the total number of escape wheel teeth, 'N' and repeat the entire analysis from the beginning of (B).

(H) - Draw the derived geometry to as large a scale as available equipment permits, in order to identify any anomalies. Verify that all designer chosen stipulations have been met. Repeat the drawing to a scale of 1:1, in order to assess practicability.

MATHEMATICAL DESIGN PROCESS
MATHEMATICAL MODEL OUTPUT

Of interest, the table below presents calculated dimensions for a geometry with same measured mean torque arm ratio (0.6837) as MS 3972/3. Also listed are measured MS 3972/3 dimensions, adjusted to a common mean torque arm, M*, of 9.94156 mm. There is fair agreement throughout, given that the available MS 3972/3 copies exhibited signs of irregular distortion.

CALCULATED PARAMETERS	**MS 3972/3 MEASUREMENTS (scaled to a common M*)**
Total number of escape wheel teeth, N = 120	120
Escape wheel pitch circle radius, R(*) mm = 67.71490	66.92 mm (to two decimal places)
Mean number of teeth spanned, n = 17.5 (52.5 degrees)	17.5 (53.2 degrees measured - distorted copy)
Angle 'a' degrees = 29.35055	30.0 degrees measured - distorted copy
Angle 'g' degrees = 13.00290	13.5 degrees measured - distorted copy
Mean torque arm ratio, t = 0.6837 (4 dec. places)	**0.6837 (4 dec. places)**
Mean torque arm, M*, mm = 9.94156	**9.94156 mm (intentional matching with M*)**
Escape arbor to esc. frame arbor, OZ, mm = 78.79334	77.98 mm (to two decimal places)
Esc. frame arbor to pallets pivot, CZ, mm = 22.26428	22.06 mm (to two decimal places)
Entry pallet locking corner to pivot, CJ, mm = 28.95195	28.83 mm (to two decimal places)
Exit pallet locking corner to pivot, AD, mm = 38.07842	38.09 mm (to two decimal places)
Pendulum arc, p, degrees = 10.11788	10.0 degrees

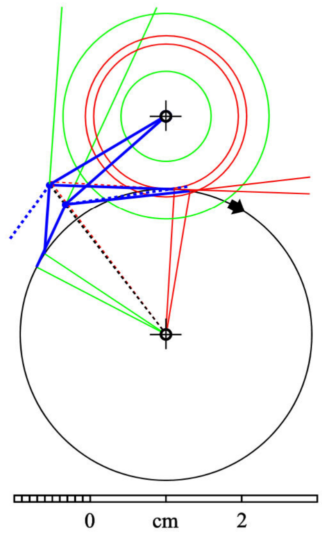
Of more immediate relevance to the creation of the perfect single pivot grasshopper escapement, the mathematical model has been used to produce a range of precisely optimised geometries, presented in increasing order of total number of escape wheel teeth, 'N'. An intentional purpose was to provide complete sets of dimensions for the benefit of clock designers who wish to avoid the process of computation.

In view of a common longcase precision regulator requirement to drive a seconds display from the escape wheel arbor, escape wheels of 30, 60, 90, 120, 150 and 180 teeth have been analysed. An unpublished study of the geometries beyond each end of that range demonstrates that there would appear to be nothing to be gained, for the present purposes at least, from greater or lesser tooth counts. A nominal pendulum arc of eleven degrees has been assumed in all cases and the optimised pendulum arcs are all close to that figure. However, desired pendulum arcs can never be precisely achieved (except by chance), by virtue of there being, in all cases, a finite number of total escape wheel teeth and mean number of escape wheel teeth spanned.

Harrison's CSM stipulations are adhered to, unless stated otherwise. Designers who choose to use the listed output must, obviously, be willing to accept Harrison's stipulations, the chosen tooth counts and the calculated pendulum arc.

Note that, by intention, no account has been taken of the variation in force along each pallet arm during impulse. The required torque ratio corrections are extremely small (see OBSERVATIONS).

As a clear easy-reference for escapement designers, each of the final designs is presented on a dedicated page, in tabular form, accompanied by a drawing of the geometry and a linear scale intended for coarse measurement only. Numerical values are presented to five decimal places, to assist those who wish to reproduce and/or confirm calculations. The escapement frame arbor and tables are placed in common locations on each page, which will enable speedy comparison, albeit at the expense of appearances. Figures should be regarded as illustrative.



NB - pallet arms not tangential to torque circles - non compliant geometry (see OBSERVATIONS).

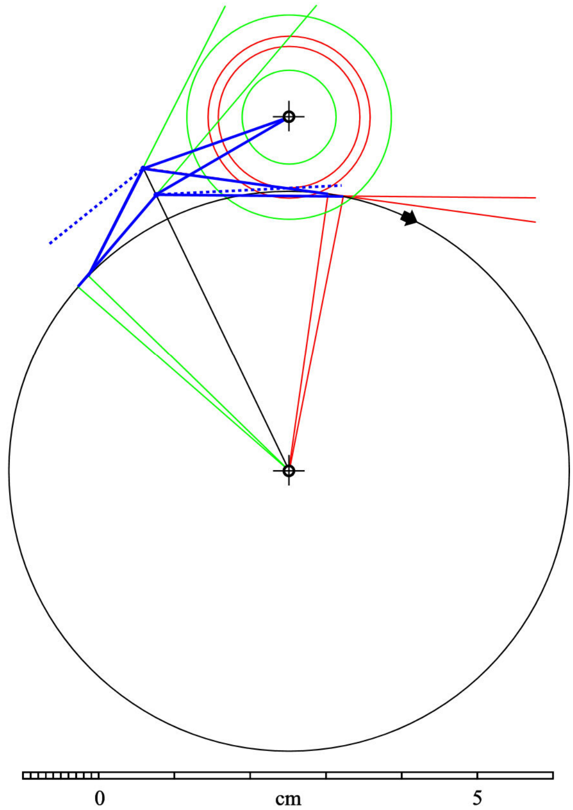
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 30
Escape wheel teeth tips pitch circle radius, R(*) mm = 19.24127
Mean number of teeth spanned, n = 5.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 40.566
Angle 'g' degrees = 3.4409
Mean torque arm ratio - not precisely achieved
Mean torque arm, M* mm - not precisely achieved

CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 28.839901
Escapement frame arbor to pallets pivot, CZ mm = 17.48822
Entry pallet locking corner to pivot, CJ mm = 9.15042
Exit pallet locking corner to pivot, AD mm = 16.47197
Entry pallet locking corner lift upon release, KV mm = 4.9289
Exit pallet locking corner lift upon release, BW mm = 0.71642
Pendulum arc, p degrees = 11.0802

FIGURE 6.44 - 30 TOOTH ESCAPE WHEEL



NB - pallet arms not tangential to torque circles - non compliant geometry (see OBSERVATIONS).

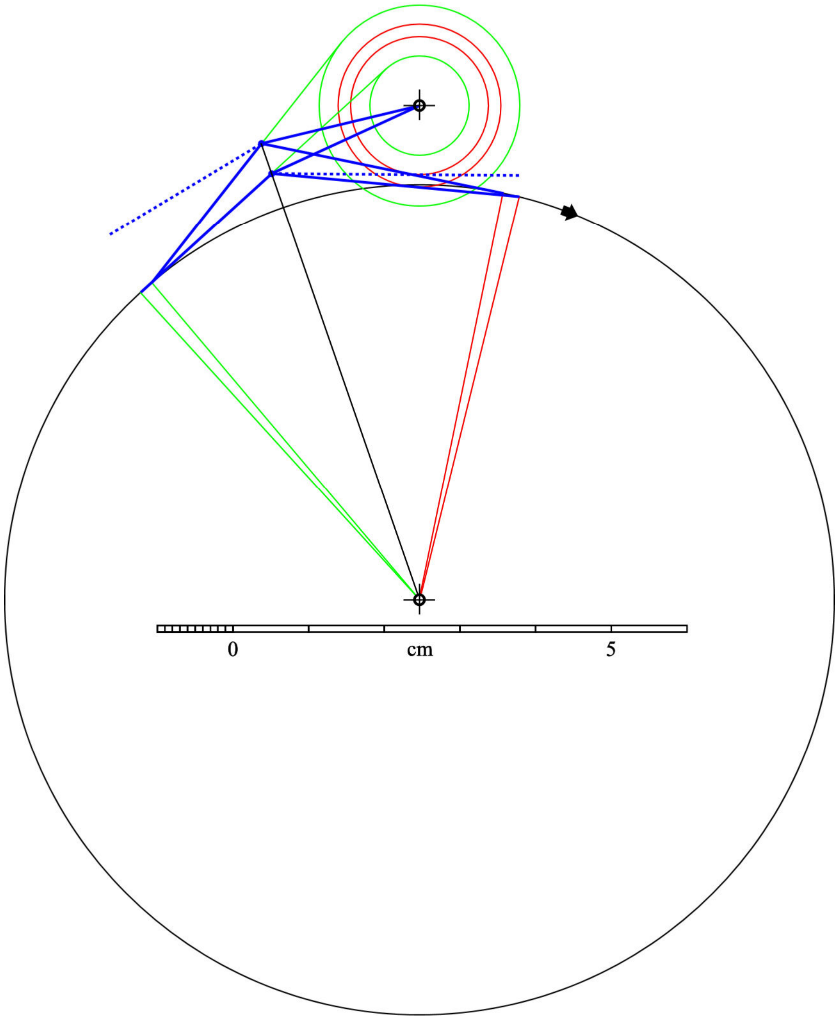
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 60
Escape wheel teeth tips pitch circle radius, R(*)mm = 36.99086
Mean number of teeth spanned, n = 9.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 33.68968
Angle 'g' degrees = 8.1133
Mean torque arm ratio - not precisely achieved
Mean torque arm, M* mm - not precisely achieved

CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 46.79175
Escapement frame arbor to pallets pivot, CZ mm = 20.29246
Entry pallet locking corner to pivot, CJ mm = 15.93868
Exit pallet locking corner to pivot, AD mm = 24.66021
Entry pallet locking corner lift upon release, KV mm = 6.60827
Exit pallet locking corner lift upon release, BW mm = 1.52645
Pendulum arc, p degrees = 10.90422

FIGURE 6.45 - 60 TOOTH ESCAPE WHEEL



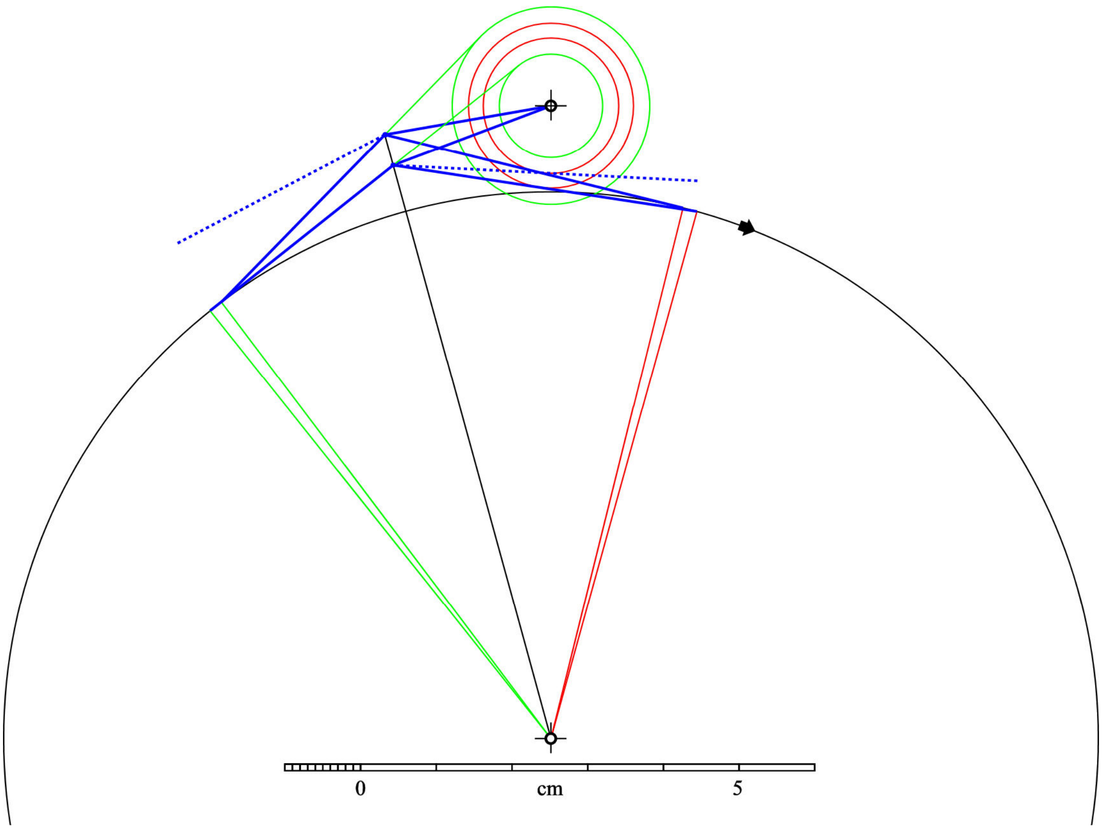
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 90
Escape wheel teeth tips pitch circle radius, R(*) mm = 54.77749
Mean number of teeth spanned, n = 13.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 30.88782
Angle 'g' degrees = 11.66846
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 65.23503
Escapement frame arbor to pallets pivot, CZ mm = 21.57127
Entry pallet locking corner to pivot, CJ mm = 23.37834
Exit pallet locking corner to pivot, AD mm = 32.76784
Entry pallet locking corner lift upon release, KV mm = 8.19443
Exit pallet locking corner lift upon release, BW mm = 2.73366
Pendulum arc, p degrees = 10.86099

FIGURE 6.46 - 90 TOOTH ESCAPE WHEEL



DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 120
Escape wheel teeth tips pitch circle radius, R(*) mm = 72.27811
Mean number of teeth spanned, n = 17.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 29.35054
Angle 'g' degrees = 13.98141
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

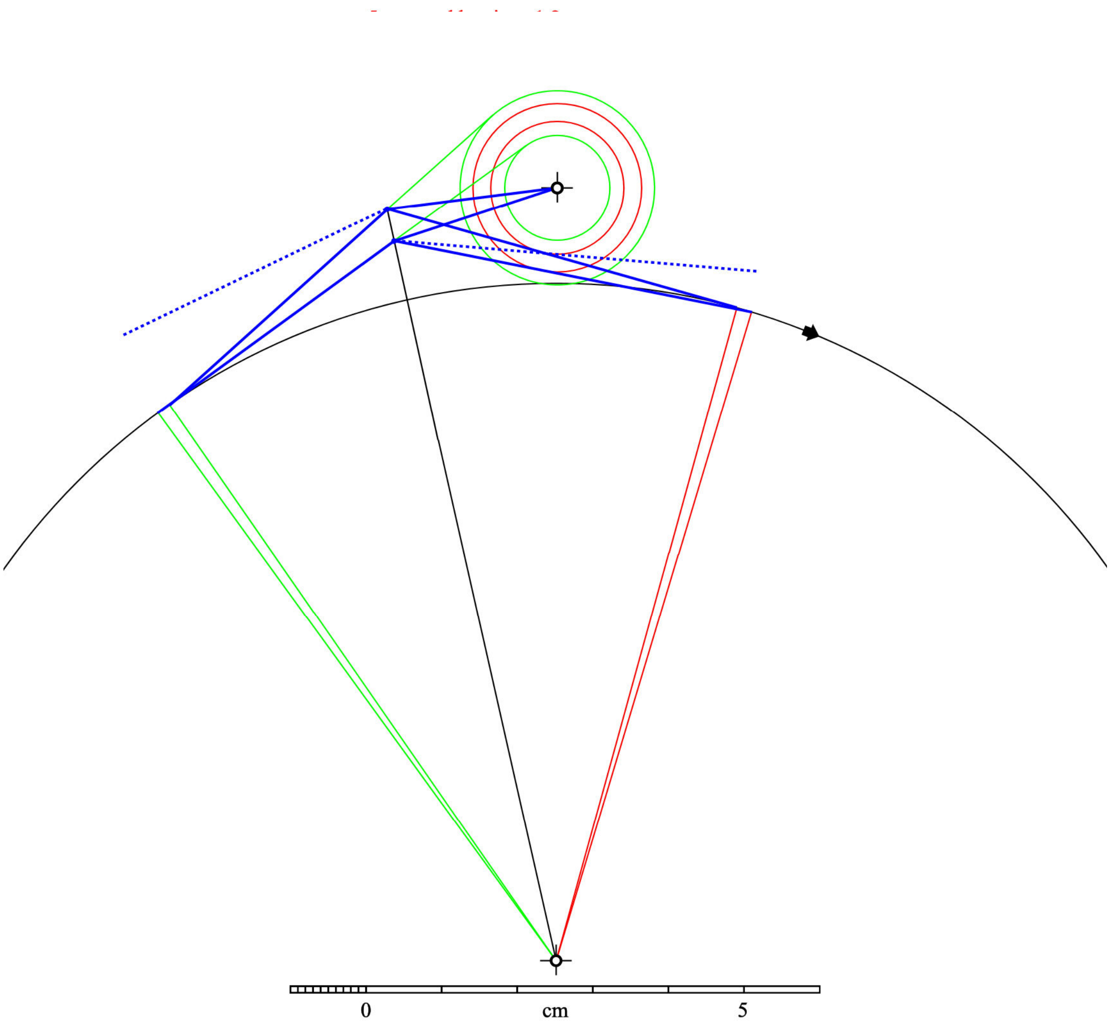
CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 83.69607
Escapement frame arbor to pallets pivot, CZ mm = 22.2813
Entry pallet locking corner to pivot, CJ mm = 30.90299
Exit pallet locking corner to pivot, AD mm = 40.64447
Entry pallet locking corner lift upon release, KV mm = 9.67033
Exit pallet locking corner lift upon release, BW mm = 4.02174
Pendulum arc, p degrees = 10.7934

SLEEPING IN OBLIVION *(see BIBLIOGRAPHY) provides sufficient constructional and dimensional detail to enable the construction of a working replica of Harrison's Final Regulator, incorporating a version of the above escapement. An arbor separation, OZ, of 3.375 inches was chosen as the controlling dimension, rather than an M* of 9.94156 mm. As a consequence, linear dimensions are consistently 2.42 % greater than the figures tabulated above, with the intentional exception of CZ, which is 0.56 % greater. For domestic settings , an alternative, reduced amplitude (narrower case) escapement is offered, incorporating a CZ 12.93 % greater than the figure tabulated above, at the expense of slight deviations from Harrison's stipulations.*

MS 3972/3 : *Calculated dimensions and measured, factored dimensions for geometry '4' of MS 3972/3 are listed at the beginning of this section. Calculated MS 3972/3 dimension OZ is 4.90273 mm less than OZ for the above geometry. Measured and factored MS 3972/3 dimension OZ is 5.71607 mm less. OZ is the most useful parameter in any comparison with Harrison's Final Regulator, being the only dimension guaranteed to be as Harrison intended. All other escapement components have been exposed to potentially incorrect incorporation, repair or replacement during attempted 'restorations' after Harrison's death. Conclusions await the release of MS 3972/3 and Final Regulator dimensions by relevant authorities, owners and/or caretakers.*

FIGURE 6.47 - 120 TOOTH ESCAPE WHEEL



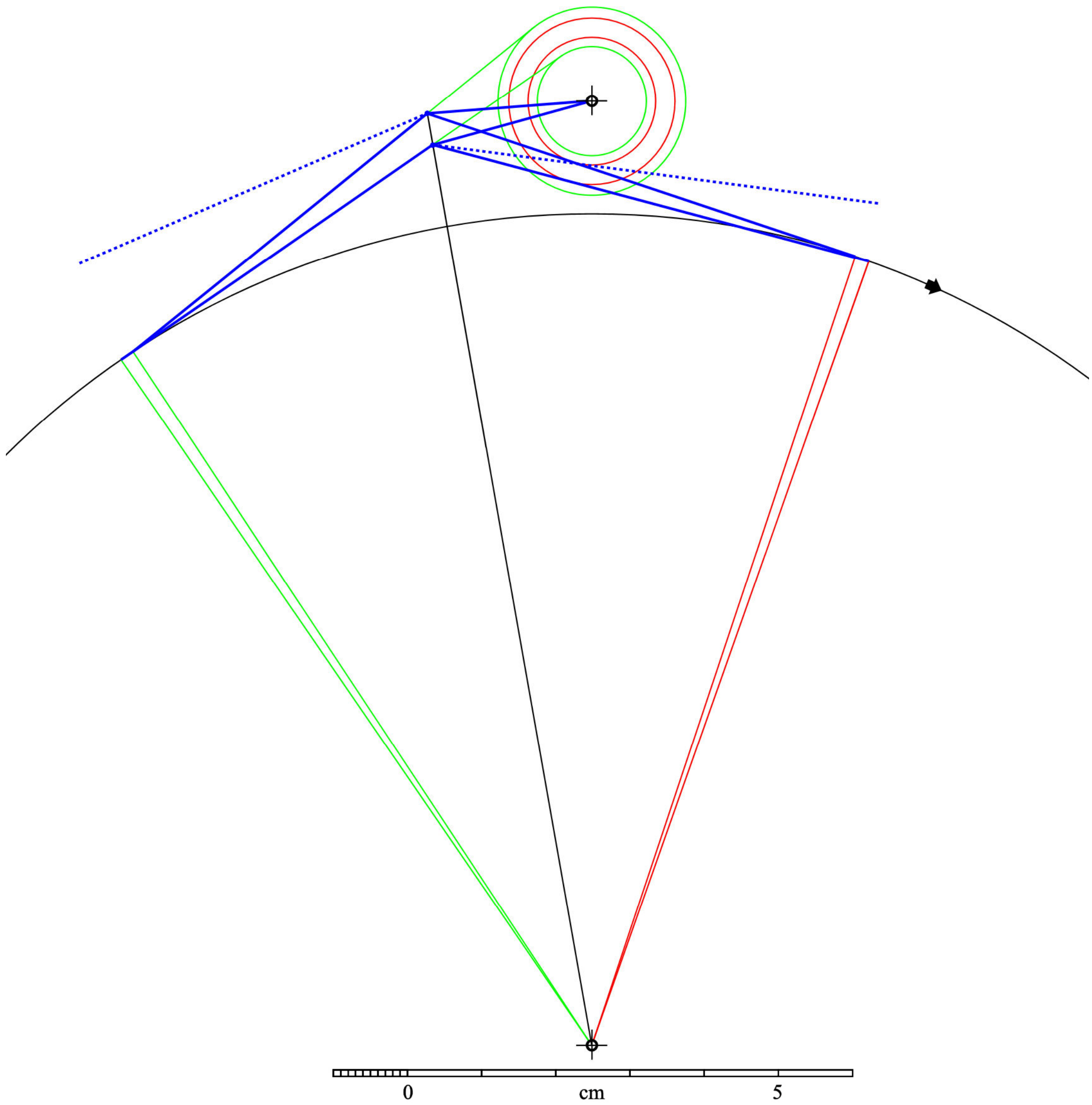
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 150
Escape wheel teeth tips pitch circle radius, R(*) mm = 89.59456
Mean number of teeth spanned, n = 21.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 28.37641
Angle 'g' degrees = 15.5797
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 102.15611
Escapement frame arbor to pallets pivot, CZ mm = 22.72634
Entry pallet locking corner to pivot, CJ mm = 38.44395
Exit pallet locking corner to pivot, AD mm = 48.39594
Entry pallet locking corner lift upon release, KV mm = 11.09071
Exit pallet locking corner lift upon release, BW mm = 5.33642
Pendulum arc, p degrees = 10.72808

FIGURE 6.48 - 150 TOOTH ESCAPE WHEEL



DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 180
Escape wheel teeth tips pitch circle radius, R(*)mm = 111.97877
Mean number of teeth spanned, n = 26.5
Equivalent pendulum length, L* mm = 994.156
Angle 'a' degrees = 28.5157
Angle 'g' degrees = 18.4806
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape arbor to escapement frame arbor, OZ mm = 127.15999
Escapement frame arbor to pallets pivot, CZ mm = 22.26392
Entry pallet locking corner to pivot, CJ mm = 50.99462
Exit pallet locking corner to pivot, AD mm = 60.83924
Entry pallet locking corner lift upon release, KV mm = 13.89113
Exit pallet locking corner lift upon release, BW mm = 8.07786
Pendulum arc, p degrees = 11.1925

FIGURE 6.49 - 180 TOOTH ESCAPE WHEEL

OBSERVATIONS

The output produced using the mathematical model leads to interesting and useful observations.

For each of the derived geometries, the choice of escapement mean teeth spanned (n) is limited to a whole number of tooth spaces, plus half a tooth space. As a consequence of such a coarse controlling parameter, some features of the geometries will exhibit slight deviations from an absolutely smooth progression.

OBSERVATIONS
IMPULSE VARIATION

By intention, no account has been taken of the variation in force along each pallet arm during impulse. The resultant torque ratio error is estimated to vary from +1.1 % (90 tooth escape wheel) to +0.3 % (180 tooth escape wheel). The errors for 30 and 60 tooth escape wheels were not investigated (see below for an explanation).

OBSERVATIONS
ACHIEVING HARRISON'S 2 TO 3 MEAN TORQUE RATIO

Two of the single pivot escapements within the chosen range could not be induced to comply precisely with Harrison's stipulation that the mean torque (arm) ratio must be two to three. Tooth counts of 30 and 60 teeth created difficulties when attempting to precisely match torque arm circles to mechanically feasible configurations. Tooth counts of 30 and 60 teeth will, therefore, be eliminated from further discussions.

Tooth counts of 90, 120, 150 and 180 all achieve a mean torque arm ratio of 2 to 3. However, as may be appreciated from a study of the four torque arm circles for each count, none of them achieve symmetrical torque arm ratios (i.e. the entry torque arm ratio is always less than the exit). A potentially valuable observation is that, as escape wheel tooth count increases, the torque arm circles, for equivalent phases of the operating cycle (i.e. either start of impulse or end of impulse), become increasingly similar, revealing a trend towards more symmetrical torque ratios.

It is especially significant that the mean torque arm (not torque arm *ratio*, please note) of the entry side is consistently extremely close to that of the exit side. That characteristic is evident from the torque circles, in that the entry torque circle pair appear to be, in all cases, symmetrically disposed relative to the exit pair. Such symmetry translates, for practical purposes, into an equality of mean entry impulse and mean exit impulse throughout the chosen range.

OBSERVATIONS
ESCAPE WHEEL SIZE

The most obvious effect of increasing the escape wheel tooth count is that there is a consistent increase in the overall size of the escapement, most especially in the diameter of the escape wheel. Harrison's most refined movement, as fitted to his Final Regulator, was large by any standards (for sound reasons, beyond the scope of this publication), which enabled a large escape wheel diameter to be accommodated with ease. More conventional movements, of lesser size, might restrict the range of feasible escapements in some applications. By virtue of Stipulation 2 (i.e. the mean torque arm must be 1/100th of the equivalent pendulum length) the escapement and escape wheel sizes are inescapably dependent upon the chosen pendulum length, which, therefore, precludes the reduction of an overly large escapement and escape wheel in an attempt to match an unsuitable movement. Of course, a shorter pendulum would resolve such issues, were it not for Stipulation 5 (i.e. a 'long pendulum' shall be incorporated). All of Harrison's pendulums were seconds beating (or close to seconds beating) and that length should be regarded as the absolute minimum.

OBSERVATIONS
TORQUE TO THE ESCAPE WHEEL

In CSM, Harrison emphasises the importance of avoiding low driving torque to the escape wheel. Unavoidable variations in friction inevitably occur throughout any movement train and their cumulative effects can be surprisingly considerable. Most especially, any slight variations in arbor pivot and gearing friction towards the top of the train (i.e. closest to the escapement) will have a greater proportional effect upon smaller escape wheel driving torques than larger ones. In that respect, as large a diameter escape wheel as possible is to be preferred, since, for a given impulse at the escape wheel tooth tips, a larger diameter escape wheel will require a higher driving torque to the escape arbor than a smaller diameter escape wheel.

OBSERVATIONS
PALLET NIB LENGTH

The length of the exit pallet nib must be chosen with care. As explained previously, the exit nib, if sufficiently long, provides some protection against trip and escape wheel runaway should torque be applied to the escape wheel with the escapement stationary, the exit pallet released from the escape wheel and the pendulum vertical. If the nib is too short, that protection will be lost. Furthermore, a longer pallet nib will offer protection from escapement trip over a greater range of pendulum operating arc than a shorter nib. However, if the exit nib is too long, it will fail to clear the path of the escape wheel tooth tips after pallet release, preventing further escape wheel rotation. Additional influences will be described shortly.

OBSERVATIONS
ESCAPE WHEEL TEETH CIRCULAR PITCH

Table 6.1 (below) demonstrates that the circular pitch of the escape wheel teeth (escape wheel tooth tips pitch circle circumference divided by the number of escape wheel teeth) is virtually constant across the chosen range of designs. The chordal pitch (shortest distance between tooth tips) unsurprisingly displays a similar degree of regularity. Such regularity in the spacing of escape wheel teeth, virtually regardless of tooth count, is of considerable practical significance, discussed in applicable sections.

No. of teeth	Radius, mm	Circular pitch, mm	Chordal pitch, mm
90	54.77749	3.824	3.821
120	72.27811	3.784	3.783
150	89.59456	3.753	3.752
180	111.97877	3.909	3.908

Table 6.1 - Demonstration of virtually constant escape wheel tooth spacing.

OBSERVATIONS
ESCAPE WHEEL TOOTH PROFILE

It is all too easy, whilst absorbed in the process of analysing pure geometries and imaginary escape wheels, to overlook a requirement for adequate escape wheel tooth mechanical strength and resistance to impact damage. As illustrated in **Figure 6.50 (next page, top illustration)**, the leading edge of each tooth must be undercut, in order to ensure contact exclusively at the pallet locking corner and to provide clearance from the pallet nib during overswing.

The trailing edge of the tooth form must remain clear of any released, descending pallet nib at all times. Those constraints define the shape and maximum size of an 'envelope' within which the escape wheel tooth profile must fit. When those envelopes are arranged around the periphery of an escape wheel, their points of overlap will define the maximum depth of the spaces between adjacent escape wheel teeth. If that depth is insufficient to accommodate the required pallet nib length, the escape wheel tooth form must be adjusted in order to provide a deeper gap. Whilst maintaining a constant tooth spacing, adjustments can only be achieved by altering the orientation of the trailing edge, as shown, exaggerated for the sake of clarity, in **Figure 6.51 (below, bottom illustration)**. That alteration weakens the tooth and, most significantly, creates a sharper, more delicate tip. Such teeth are more susceptible to damage should the escapement trip and the escape wheel runaway.

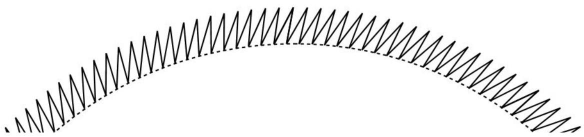


Figure 6.50

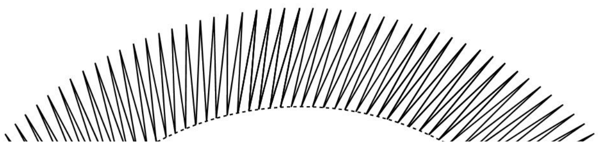


Figure 6.51

OBSERVATIONS
ESCAPE WHEEL RECOIL

For a constant escape wheel tooth circular pitch (see Table 6.1), the amount of recoil, in terms of movement at the tooth tips, will also be constant, to all intents and purposes. Although a larger escape wheel will, therefore, recoil through a smaller angle, recoil of the associated train will be undiminished. However, recoil has already been identified as irrelevant, for all practical purposes, when used in combination with Harrison's almost frictionless type of train, or a common train fitted with a device called an 'escape arbor remontoire', which completely isolates the escape arbor from the train. Such devices are vital to consistent performance.

OBSERVATIONS
PALLET LOCKING CORNER LIFT UPON RELEASE

Care has been taken to calculate pallet locking corner lifts, KV and BW, for those parameters are of considerable influence.

In particular, as explained in the PALLET NIB LENGTH section, the exit pallet lift and the length of the exit pallet nib are important features. Exit pallet lift, BW, becomes markedly greater with increasing escape wheel tooth count. It therefore follows that, for maximum trip protection to be maintained, the exit pallet nib length must also increase as escape wheel tooth count increases. The ESCAPE WHEEL TOOTH PROFILE section explained how escape wheel tooth mechanical strength and resistance to impact are diminished as pallet nib length is increased. Other factors aside, that would suggest that the escapement geometry generating minimum exit pallet nib lift will require the shortest exit pallet nib length, the shallowest gap between adjacent escape wheel teeth and, therefore, the strongest escape wheel tooth profile. Much depends upon the extent to which the designer is willing to trade trip protection against escape wheel tooth strength.

OBSERVATIONS
CHOICE OF ESCAPE WHEEL TOOTH COUNT

The designer faces a difficult decision as to the optimum number of escape wheel teeth to be incorporated, for much depends upon the nature of each individual application. For his Final Regulator, as described in CSM, Harrison was emphatic that a 120 tooth (4 mins) escape wheel should be used. The movement of that regulator was of considerable size and could accommodate such an escape wheel and escapement with ease.

Supporting Harrison's choice in all situations is far from straightforward. The observations listed herein do not all lead in the same direction, nor do they all converge towards a single solution. As is typical of any design process, a great deal depends upon the demands of the intended timepiece and the willingness of the designer to accept compromises. Trip protection, escape wheel tooth strength, escape wheel tooth tip impact resistance, escapement size etc. must all be considered. In this particular case, it is fortunate that practical research and experience with a near-replica of Harrison's Final Regulator confirms that his choice of a 120 tooth escape wheel provides an escapement with excellent overall characteristics in that application. The exit pallet nib lift is compatible with the escape wheel tooth pitch in terms of trip protection, a quite robust tooth form can be accommodated and, most importantly, a relatively high torque to the escape wheel arbor is demanded..

For more conventional movements, of considerably lesser size, difficulties might arise as a consequence of the large escape wheel diameter demanded by 120 teeth, in which case the only viable, smaller option would be the 90 tooth configuration. A 90 tooth escape wheel would be almost 110mm diameter, which would still require careful incorporation into a conventional movement, although mounting of the escape wheel to the rear of the rear main plate is a possible solution. The most significant disadvantage of a 90 tooth escapement is a reduced driving torque to the escape wheel when compared to wheels of greater tooth counts.

The 150 and 180 tooth escape wheel configurations are extremely large, by any standards and the exit pallet nib lift for 180 teeth raises concerns, in terms of accommodating an ideal exit pallet nib length. In terms of consistent performance, the considerable torque demanded at the escape wheel arbor is an indisputable advantage.

OBSERVATIONS
MECHANICAL CONSIDERATIONS

The objective during nib capture is to present the nib locking corner to the applicable escape wheel tooth tip with absolute precision. It is, therefore, vital that all active dimensions be manufactured to the closest possible tolerances. All pivots and arbors must have no 'shake' ('play'), but must be sufficiently free that they offer no resistance to rotation.

Pallet nib locking corners must be extremely sharp and precisely perpendicular to the plane of the escape wheel. Escape wheel tooth tips must also be sharp, although a *very slight* flat, tangential to the pitch circle, will reduce any tendency to deform, whilst improving resistance to damage should the escapement trip. The pitch circle diameter must not be reduced by formation of the slight flat, which will demand some thought and care when the teeth are initially formed. A rounded end, however slight, must be avoided, being insecure during the capture phase of operation, when the generation of adequate static friction is demanded.

The choice of materials is an important contributor to successful operation and longevity. Lignum vitae is a dense, naturally greasy wood, offering excellent wear resistance and low friction. It was used extensively by Harrison as a maintenance-free bearing material, with a potential life span measured in centuries. As such, it is an ideal bearing material at the pallet arms pivots. However, lignum vitae is not suitable as a pallet nib material, low friction being the complete opposite of what is required, demanding unnecessarily high escape wheel torque for reliable capture. Compromise hardwoods, such as hard oak, will offer useful service and will enable pallet arms to be manufactured as a single piece. Caution is required, however, in view of the tendency for oak to induce corrosion of certain materials. For example, the (non original) pallet arm balance weights of Harrison's Final Regulator, manufactured in lead, have required replacement due to severe corrosion. The resistance of chosen materials to damage during escape wheel runaway is also an important consideration, although damage to the pallet nibs is preferable to damage to the escape wheel.

A correctly constructed, installed and adjusted grasshopper escapement will offer centuries of continuous, maintenance free operation, by virtue of there being, to all intents and purposes, absolutely no sliding friction or wear.

ELIMINATING ERRORS

The following sections will consider all potential causes of error relating to the single pivot grasshopper escapement.

As will be demonstrated, grasshopper escapement errors are, in most cases, markedly different to those of the anchor or dead beat. In fact, in most cases, errors are totally non-existent, as will be explained. Nevertheless, it will be instructive to repeat the topic headings presented for the anchor and dead beat escapements and apply their intent to the single pivot grasshopper. The purpose is to demonstrate the remarkably numerous performance advantages of the grasshopper escapement and, in the process, enable an appreciation of Harrison's remarkable achievement.

ELIMINATING ERRORS
RECOIL

The single pivot grasshopper escapement is a recoil escapement. By virtue of the large pendulum amplitude and energy, recoil can be quite marked. Nevertheless, there are no adverse effects for the escapement itself. There is, to all intents and purposes no sliding friction, no wear and, therefore, no increase in wear due to the high loads often generated by recoil.

The effects of grasshopper recoil upon a conventional train are no better than those produced by the anchor and are, clearly, worse than those of the dead beat. Solutions include the abandonment of the common form of train and the adoption of a Harrison train, which functions with virtually no sliding friction and an extremely low resistance to forward or, most significantly, reverse rotation. Another extremely useful option is the fitment of an escape arbor remontoire. In simplified terms, a remontoire isolates the escape wheel (and, therefore, the escapement and pendulum) from the movement train. Torque delivery to the escape arbor is rendered either extremely constant, or, at worst, cyclically regular and recoil of the escape wheel is absorbed by the remontoire, rather than transmitted to the train.

ELIMINATING ERRORS
CIRCULAR ERROR

In complete contrast to the philosophies of the anchor and dead beat escapements, Harrison specifically demands that the amplitude of the pendulum must be large. One feature of the grasshopper escapement is that it is capable of generating such large arcs with no adverse effects. Harrison specifies a maximum arc of fifteen degrees.

What Harrison has done is typically confident and bold, born of establishing honest truths and adopting straightforward thinking. He was aware that undesirable disturbances to the motion of a pendulum would have a proportionally greater effect on the regular motion of the pendulum if the pendulum was possessed of low energy, rather than high energy. He therefore advocated a pendulum with high energy, which he achieved by incorporating an escapement geometry producing a large amplitude. He was, therefore, consciously permitting considerable circular error, which he then tamed by fitting effective pendulum suspension cheeks, to his own, unique specifications.

Harrison determined that cycloidal cheeks in combination with the characteristics of his early grasshopper escapement tended to provide unsatisfactory compensation and that slightly more 'open' cycloidal cheeks seemed to be required. When he introduced his stipulation that the mean torque ratio should be 2 to 3, he would appear to have discovered, no doubt by practical experimentation, that the cycloid, whether 'open' or not, was no longer valid and that cheeks in the form of a remarkably simple circular arc were appropriate. It can only be concluded that the 2 to 3 torque ratio is, to some extent, negating circular error.

ELIMINATING ERRORS
ERROR DUE TO WEAR

As described earlier, the single pivot grasshopper escapement operates without any sliding friction whatsoever, apart from an insignificant rotation at the pallets pivot. Pallets pivots would appear to have a working life measured in centuries, provided that materials are chosen sensibly (brass pivots running in lignum vitae bushes being an ideal, maintenance free combination). Static friction at the pallet nib locking corners cannot produce wear, although impact between those corners and the escape wheel tooth tips often produces an insignificant witness mark on newly constructed wooden pallets, which stabilises, once formed and may, for all practical purposes, be ignored.

Put simply, there is no wear. There cannot, therefore, be any error due to wear.

ELIMINATING ERRORS
LUBRICATION ERROR

For escapement configurations such as the anchor and dead beat, it is necessary to apply lubricant in order to reduce friction and wear to acceptable levels. Unfortunately, clock oils, be they of animal, mineral, vegetable or synthetic origin, are subject to evaporation, oxidation, contamination, molecular modification etc. leading to a degradation in their wear reducing properties. Of considerable significance to the generation of errors, such lubricant deterioration will also alter the magnitude of frictional forces between relevant components.

In complete contrast, the single pivot grasshopper escapement generates no sliding friction. There is, therefore, no requirement for lubrication at any point of the escapement, thereby completely eliminating that source of error.

In addition, Harrison's movements were, by ingenious design, entirely free from sliding friction, apart from the insignificant pivoting of lignum vitae rollers and hubs upon fine brass axles. Train lubrication was thereby entirely eliminated, as are any errors that it would otherwise introduce.

ELIMINATING ERRORS
ESCAPEMENT ERROR

The most ideal timekeeping (ignoring the use of evacuated, temperature regulated enclosures) will be achieved by a completely free-swinging pendulum. Unfortunately, there is a requirement to add energy to the pendulum, in order to sustain its motion. In addition, the number of pendulum swings must be counted, in order to measure the passage of time. An effect associated with adding energy and counting pendulum swings by mechanical means via an escapement is that, no matter how delicately we count or how carefully we add energy, we will inevitably interfere with the otherwise free swing of the pendulum, altering its period. Any alteration in the period of the pendulum caused by the escapement is called escapement error. For common escapements, such as the anchor and dead beat, the degree of escapement error will depend upon how constant the torque supplied to the escape wheel is, how the impulse from the escape wheel to the pallets varies, how friction alters during each cycle and over many cycles, how consistent the applied lubricants are, how much of pendulum arc the impulse, drop and recoil events occupy and at what positions of the pendulum arc they occur.

The next section will consider, in detail, the causes of escapement error in the single pivot grasshopper escapement.

ELIMINATING ERRORS
ESCAPEMENT ERROR
CAUSES OF ESCAPEMENT ERROR

Before describing the causes of escapement error, we should acknowledge that many other sources of error affect timekeeping performance. We shall, however, confine our attentions to the escapement.

(A) - Torque supplied to the escape wheel.

The driving weight supplies energy to the movement train, which emerges at the escape wheel as a torque about the escape arbor and thence as a force at the engaged escape wheel tooth tip.

As already mentioned, Harrison's movements required absolutely no lubrication. His virtually frictionless gearing did, however, generate marked variations in torque delivery to the escape arbor. Harrison eliminated those variations by fitting his own, typically ingeneous, frictionless, high performance, remontoire, rendering torque to the escape arbor virtually constant.

(B) - Torque transmitted by the pallets.

By virtue of frictionless and lubricant-free operation, the translation of escape arbor torque to torque at the escapement frame arbor, and thence to the crutch and the pendulum, occurs without variation. Apart from slight impact marking at the locking corners of new pallets, which rapidly stabilises, there is no wear to the escapement or escape wheel. As a consequence, the geometry is perfectly stable and the single pivot grasshopper escapement exhibits unaltered torque transmission characteristics throughout centuries of operation.

(C) - Effect of variations in drop.

Previous studies of escapement error for the anchor and dead beat escapements have clearly identified and explained how lubricant degradation in combination with escape wheel drop is a significant cause of unpredictable variations in escapement error. The single pivot grasshopper escapement completely eliminates those sources of error, by virtue of not only operating without lubricant, but also, astonishingly, entirely without drop. The escape wheel is never free from the escapement, since one escape wheel tooth is always engaged with a pallet locking corner and pallet capture and release events are, effectively, simultaneous. If there is no lubrication or drop, then there can be absolutely no escapement error due to lubrication or drop and, therefore, no associated variations in escapement error. As a result, although continuous pallet engagement with the escape wheel is as far removed from the ideal of a completely free pendulum as it could possibly be, it achieves far superior performance, when compared to escapements incorporating phases of freedom.

(D) - Effect of variations in pendulum amplitude.

Variations in pendulum amplitude will be absorbed by the escapement in the form of variations in escape wheel recoil, for no other phase of the operating cycle can vary.

The grasshopper escapement incorporates recoil without generating errors. Recoil commences at the point of pallet nib locking corner capture and advances no further when the motion of the pendulum ceases at the extremity of is motion. Variations in the extremity of swing of the pendulum are of no consequence, except for their effect upon circular error. Circular error was discussed in an earlier section.

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